H

National
Qualifications

X847/76/11

Mathematics
Paper 1 (Non-calculator)

Duration — 1 hour 15 minutes

Total marks — 55

SECTION 1 — 44 marks

Attempt ALL questions.

SECTION 2 — 11 marks

Attempt EITHER Part A OR Part B.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\begin{aligned} \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

SECTION 1 — 44 marks

Attempt ALL questions

1. Find the value of k for which the equation $kx^2 + 3x - 4 = 0$ has equal roots. **3**

2. Given that $f(x) = (x^2 + 1)^5$, find $f'(1)$. **3**

3. A function $f(x)$ is defined on $\mathbb{R}$, by

$$f(x) = \frac{x+3}{2}.$$

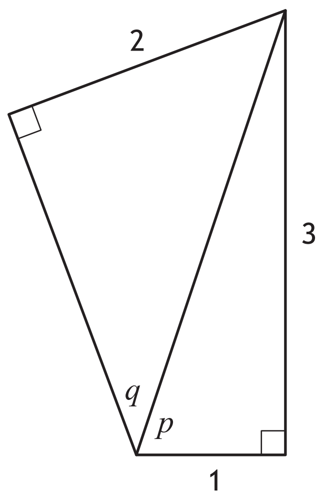
Find the inverse function, $f^{-1}(x)$. **3**

4. Determine whether the line passing through (–4, 2) and (2, –7) is perpendicular to the line with equation $3y = 2x + 9$. **3**

[Turn over

MARKS

5. Two right-angled triangles are shown below.



(a) Determine the value of

(i) $\sin p$ **1**

(ii) $\cos q$. **2**

(b) Find the exact value of $\cos(p+q)$. **3**

6. Functions f and g are defined on $\mathbb{R}$ by

- $f(x) = 2x + 5$
- $g(x) = x^2 - 2x$.

(a) Find an expression for $f(g(x))$. **2**

(b) Find an expression for $g(f(x))$. **1**

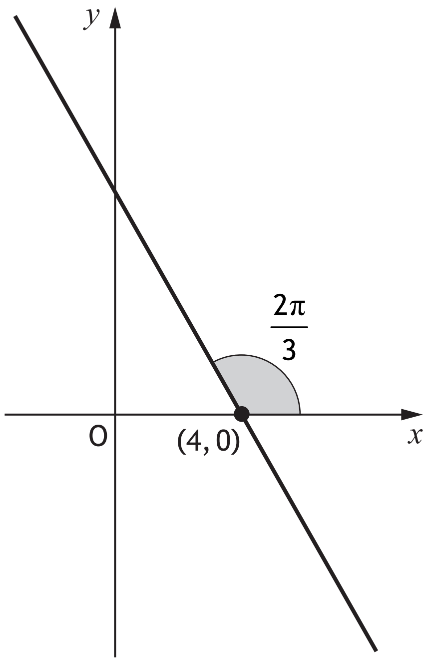
(c) Express $g(f(x)) - f(g(x))$ in the form $a(x+b)^2 + c$. **4**

MARKS

7. Find $\int 6\cos\left(3x+\frac{\pi}{4}\right)dx$. **2**

8. A line makes an angle of $\frac{2\pi}{3}$ with the positive direction of the x-axis.

It passes through the point $(4, 0)$.

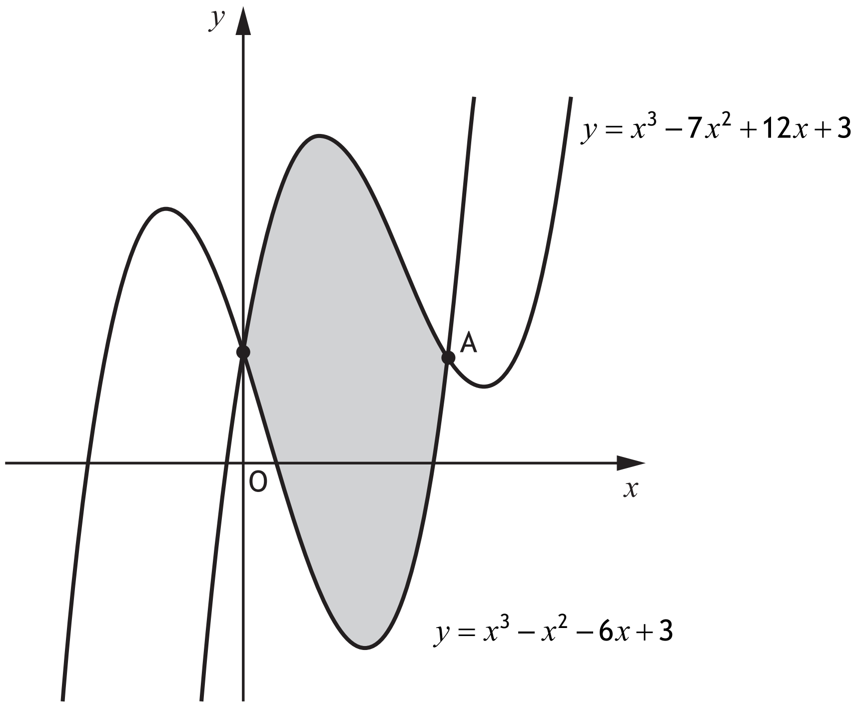


Determine the equation of the line. **3**

[Turn over

9. The diagram shows the curves with equations $y = x^3 - 7x^2 + 12x + 3$ and $y = x^3 - x^2 - 6x + 3$.

The curves intersect on the y-axis and at point A.



(a) Find the x-coordinate of A. **2**

(b) Calculate the shaded area. **5**

10. Factorise $6x^3 - 13x^2 + 4$ fully. **4**

MARKS

11. A function, f, defined on $\mathbb{R}$, is such that

- the maximum value of f is 8
- the maximum occurs when $x = 6$.

The function g is given by $g(x) = 2f(x) - 9$.

(a) State the maximum value of g. **1**

The function h is given by $h(x) = f(x-4) + 5$.

(b) (i) State the maximum value of h. **1**

(ii) State the value of x when the maximum value of h occurs. **1**

[END OF SECTION 1]

[Turn over

MARKS

SECTION 2 — 11 marks

Attempt EITHER Part A OR Part B

Part A

12. Points A, B, and C are collinear, with B dividing AC.

- A has coordinates $(4, 2, -5)$
- B has coordinates $(7, -4, 1)$
- $|\overrightarrow{BC}| = 6$

(a) (i) Find $|\overrightarrow{AB}|$. **2**

(ii) State the ratio in which B divides AC. **1**

(b) Determine the coordinates of C. **1**

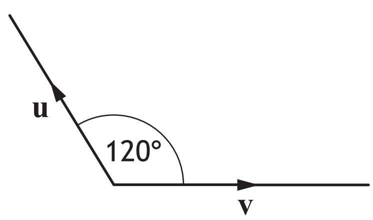
13. A sequence is generated by the recurrence relation $u_{n+1} = \frac{2}{3}u_n + 8$, $u_7 = 20$.

(a) Determine the value of u_5. **2**

This sequence approaches a limit as $n \to \infty$.

(b) Determine the limit of this sequence. **2**

14. The angle between vectors $\mathbf{u}$ and $\mathbf{v}$ is 120°.
$|\mathbf{u}| = 4$ and $|\mathbf{v}| = 5$.



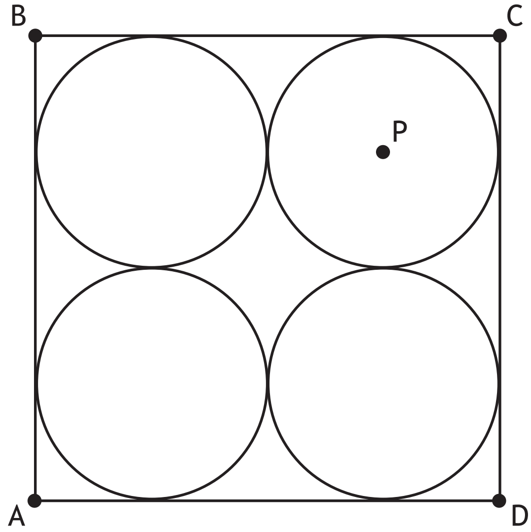
Calculate $\mathbf{u}.(\mathbf{u} + \mathbf{v})$. **3**

MARKS

Part B

15. ABCD is a square containing four congruent circles.

A is the point (2, 1), and D is the point (10, 1).



Determine the equation of the circle with centre P. **3**

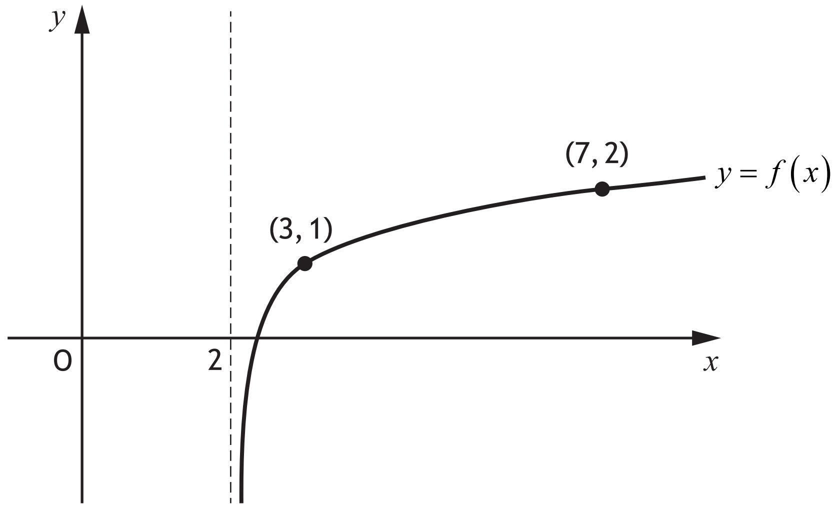
16. Evaluate $\log_2 6 + \log_2 12 - 2\log_2 3$. **4**

[Turn over

MARKS

17. A logarithmic function, f, is defined for $x > 2$.

The diagram shows the graph of $y = f(x)$.



The inverse function, $f^{-1}(x)$, exists.

(a) On the diagram in your answer booklet, sketch the graph of the inverse function. **2**

(b) Given that $f(x) = \log_5(x-2)+1$, find the coordinates of the point where the graph of $f^{-1}(x)$ crosses the y-axis. **2**

[END OF SECTION 2]

[END OF QUESTION PAPER]

H

National Qualifications

X847/76/12

Mathematics Paper 2

Duration — 1 hour 30 minutes

Total marks — 65

SECTION 1 — 52 marks

Attempt ALL questions.

SECTION 2 — 13 marks

Attempt EITHER Part A OR Part B.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

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* X 8 4 7 7 6 1 2 *

MARKS

SECTION 1 — 52 marks

Attempt ALL questions

1. Determine the equation of the tangent to the curve $y = 2x^3 - 8x^2 + 14$ at the point where $x = 3$. **4**

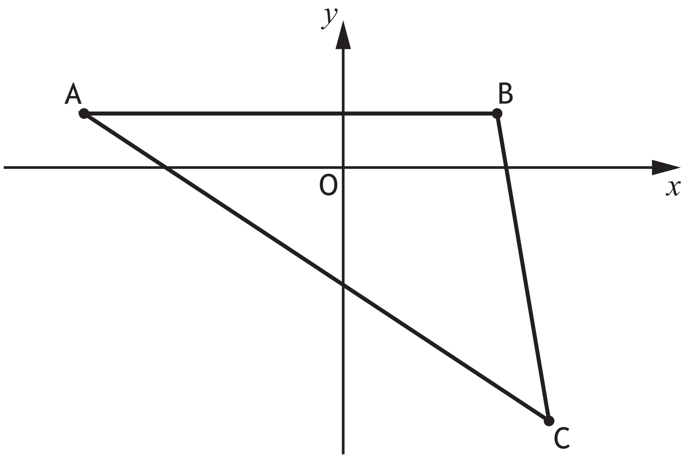
2. Find $\int \frac{6}{(x+5)^{\frac{3}{2}}}\,dx$, $x > -5$. **3**

3. Given $h(t) = \sin\left(2t + \frac{\pi}{6}\right)$, determine the rate of change of h when $t = 10$. **3**

[Turn over

MARKS

4. Triangle ABC has vertices A (–5, 1), B (3, 1) and C (4, –5).



(a) The line L_1 is the altitude through B.

Find the equation of L_1. **3**

(b) The line L_2 is the perpendicular bisector of AB.

Find the equation of L_2. **3**

(c) Determine the coordinates of the point of intersection of L_1 and L_2. **1**

5. (a) Express $3\cos t° + 5\sin t°$ in the form $k\sin(t + a)°$, $k > 0$, $0 < a < 360$. **4**

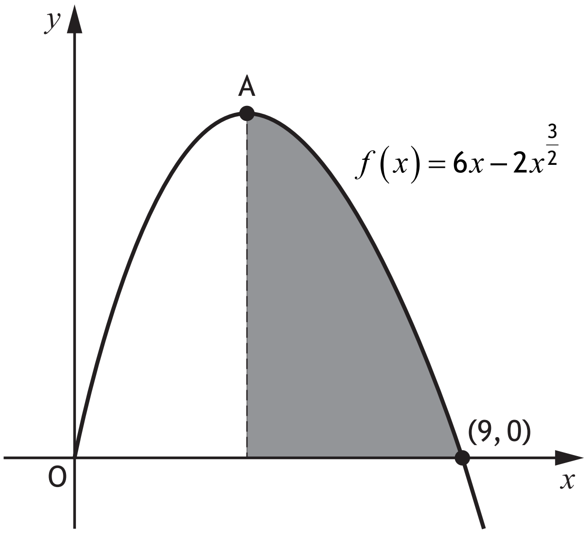
(b) A function, f, is defined by $f(t) = 3\cos t° + 5\sin t°$, $0 \leq t < 360$.

(i) State the minimum value of $f(t)$. **1**

(ii) Determine the value of t where this minimum occurs. **1**

6. The graph of the function $f(x) = 6x - 2x^{\frac{3}{2}}$, $x \geq 0$ is shown.

The point A is a stationary point of $f(x)$.

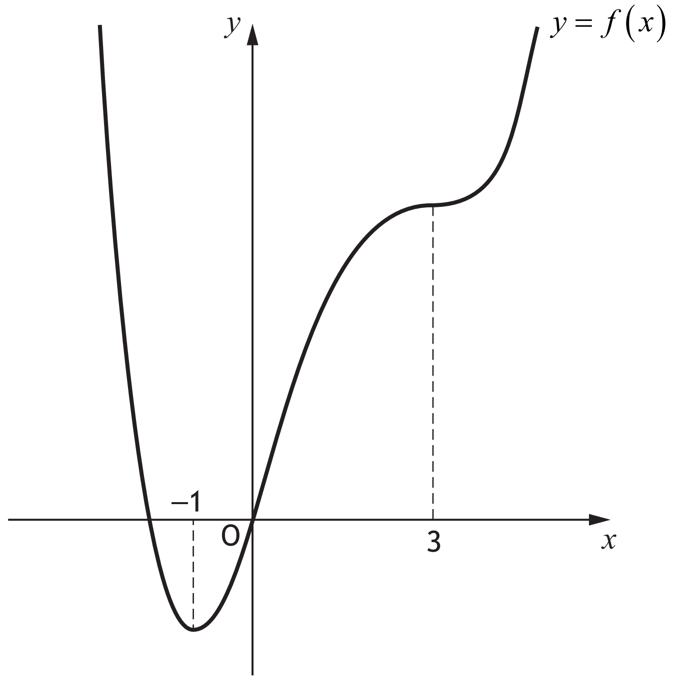


(a) Determine the x-coordinate of the stationary point A. **3**

(b) Hence calculate the shaded area. **4**

[Turn over

7. The diagram shows the graph of $y = f(x)$, which has stationary points at $x = -1$ and $x = 3$.



On the diagram in your answer booklet, sketch a possible graph of $y = f'(x)$. **3**

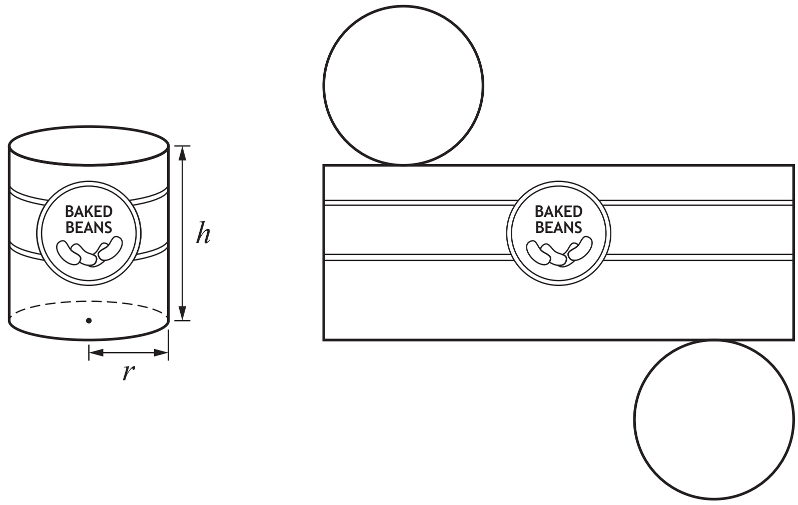
8. Solve the equation $2\sin(3x - 60)° + 1 = 0$, $0 \leq x < 180$. **4**

MARKS

9. A cylindrical tin of baked beans has a volume of 450 cm^3.

The radius of the tin is r cm and its height is h cm.

A net of the tin is shown in the diagram.



(a) Show that the surface area of the tin, A square centimetres, is given by

$$A(r) = 2\pi r^2 + \frac{900}{r}.$$ **3**

(b) Determine the radius that will minimise the surface area. **6**

10. (a) Show that $2\tan x\cos^2 x = \sin 2x$, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$. **2**

(b) Given that

- $\frac{dy}{dx} = 6\tan x\cos^2 x$, and
- $y = 3$ when $x = 0$,

express y in terms of x. **4**

[END OF SECTION 1]

[Turn over

MARKS

SECTION 2 — 13 marks

Attempt EITHER Part A OR Part B

Part A

11. (a) Given A(3, 1, 8), B(–2, 5, 1) and C(7, –6, 3), express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. **2**

(b) Hence calculate the size of angle BAC. **4**

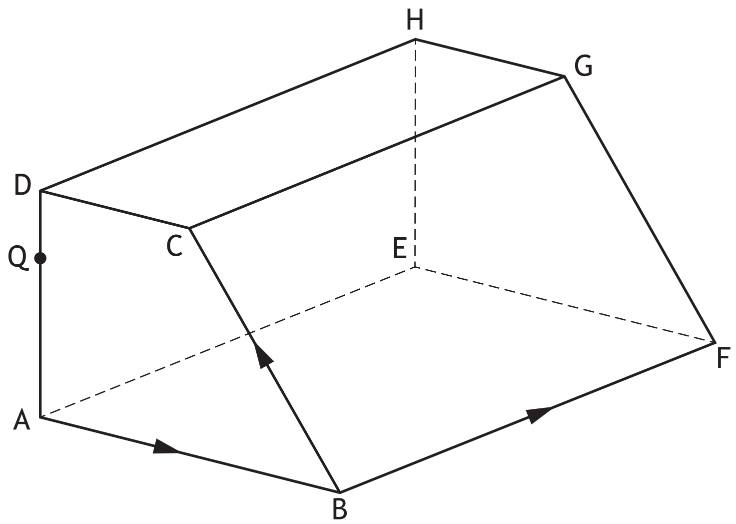
12. A sequence of real numbers is such that

- the terms of the sequence satisfy the recurrence relation $u_{n+1} = 9u_n - 440$
- $u_{n+1} > u_n$ for all values of n.

The difference between two particular terms, u_{k+1} and u_k, is 1000.

Determine, algebraically, the value of u_k. **3**

13. ABCD,EFGH is a prism.



- $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -4 \\ 6 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} -7 \\ 5 \\ 3 \end{pmatrix}$ and $\overrightarrow{BF} = \begin{pmatrix} 7 \\ 11 \\ -2 \end{pmatrix}$.
- $\overrightarrow{AB} = 2\overrightarrow{DC}$.

(a) Express $\overrightarrow{CF}$ in component form. **1**

(b) Hence, or otherwise, express $\overrightarrow{DF}$ in component form. **1**

(c) The point Q lies on the line AD.

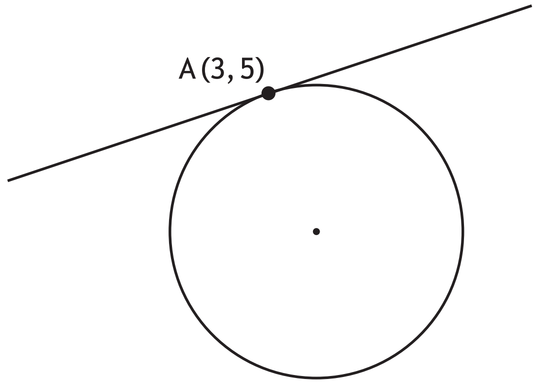
Given that $\overrightarrow{QF} = \begin{pmatrix} 17 \\ 5 \\ 0 \end{pmatrix}$, find $\overrightarrow{QD}$. **2**

[Turn over

MARKS

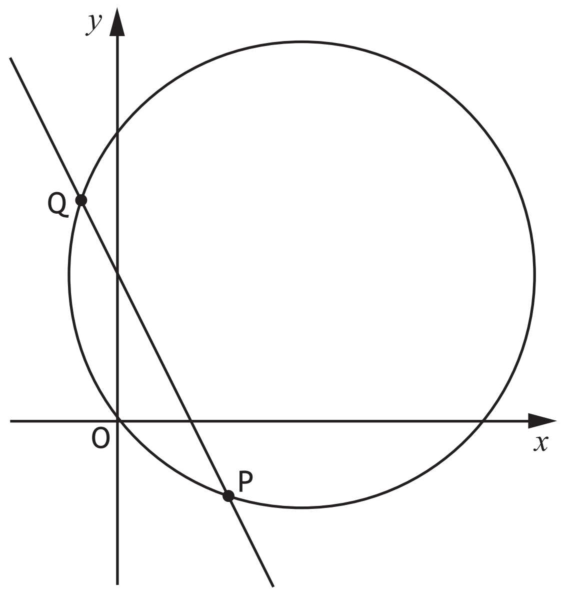
Part B

14. The point A (3, 5) lies on the circle with equation $x^2 + y^2 - 10x + 2y - 14 = 0$.



Find the equation of the tangent to the circle at A. **4**

15. The line $y = 4 - 2x$ intersects the circle $x^2 + y^2 - 10x - 8y + 1 = 0$ at the points P and Q.

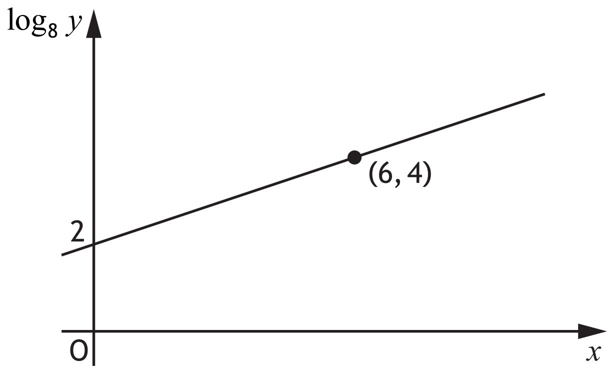


Find the coordinates of the points of intersection. **4**

MARKS

16. Two variables, x and y, are connected by the equation $y = ab^x$.

The graph of $\log_8 y$ against x is a straight line as shown.



Find the values of a and b. **5**

[END OF SECTION 2]

[END OF QUESTION PAPER]