2007 Mathematics

Higher – Paper 1

Finalised Marking Instructions

1.01

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.01		3	G2, G3	CN	7063	1		2	3		

Find the equation of the line through the point $(-1, 4)$ which is parallel to the line with equation $3x - y + 2 = 0$. **3**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	express in standard form
$\bullet^2$	ic	interpret gradient
$\bullet^3$	ic	state equation of line

Primary Method : Give 1 mark for each •

$\bullet^1$	$y = 3x....$	stated/implied by $\bullet^2$
$\bullet^2$	$gradient = 3$	stated/implied by $\bullet^3$
$\bullet^3$	$y - 4 = 3(x - (-1))$	
or		
$\bullet^1$	$form\ is\ 3x - y + c = 0$	
$\bullet^2$	$3 \times (-1) - 4 + c = 0$	
$\bullet^3$	$c = 7$	

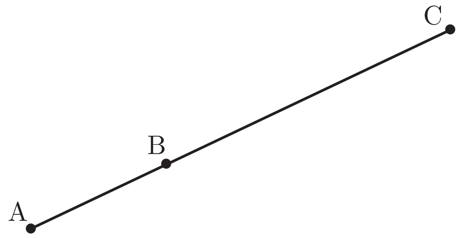
Notes

1 Accept any form of the answer (with or without working) for 3 marks

1.02

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.02		4	G17	CN	7001	1	1	2	4		

Relative to a suitable coordinate system A and B are the points $(-2,1,-1)$ and $(1,3,2)$ respectively.
A, B and C are collinear points and C is positioned such that BC = 2AB.
Find the coordinates of C. **4**



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$\bullet^1$	ss	introduces vectors
$\bullet^2$	pd	completes
$\bullet^3$	ic	interprets positions
$\bullet^4$	ic	finds C

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$ $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$ stated or implied by $\bullet^2$

$\bullet^2$ $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

$\bullet^3$ $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix}$

$\bullet^4$ $C = (7,7,8)$

Notes

1 Treat C=$\begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix}$ as bad form

2 In Alt. method 2, without a diagram only $\bullet^2$, $\bullet^3$ and $\bullet^4$ are available.

Alt. method 1

$\bullet^1$ $\boldsymbol{c} - \boldsymbol{b} = 2\boldsymbol{b} - 2\boldsymbol{a}$

$\bullet^2$ $\boldsymbol{c} = 3\boldsymbol{b} - 2\boldsymbol{a}$

$\bullet^3$ $\boldsymbol{c} = 3\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} - 2\begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$

$\bullet^4$ $C = (7,7,8)$

Alt. method 2

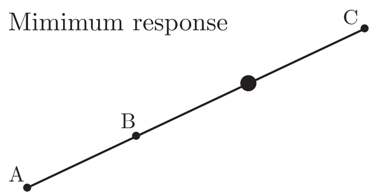
$\bullet^1$ *ic* *diagram* $\rightarrow\rightarrow$

$\bullet^2$ *pd* $x = 7$

$\bullet^3$ *pd* $y = 7$

$\bullet^4$ *pd* $z = 8$

Mimimum response



1.03

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.03	a	2	A4	CN	7069	1		1	2		
	b	2	A4			1		1	2		

Functions f and g, defined on suitable domains, are given by

$f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find (a) $g\big(f(x)\big)$ **2**

(b) $g\big(g(x)\big)$ **2**

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$\bullet^1$	ss	know to start from the "inside"
$\bullet^2$	ic	interpret composite function
$\bullet^3$	ss	know to start from the "inside"
$\bullet^4$	ic	interpret composite function

Primary Method : Give 1 mark for each •

$\bullet^1$	$g\big(f(x)\big) = g(x^2 + 1)$	s/i by $\bullet^2$
$\bullet^2$	$1 - 2(x^2 + 1)$	
$\bullet^3$	$g(g(x)) = g(1 - 2x)$	s/i by $\bullet^4$
$\bullet^4$	$1 - 2(1 - 2x)$	

Notes

1 in (a) :

for finding $f(g(x))$:

$g(1-2x)$ *no mark*

$(1-2x)^2 + 1$ *award* $\bullet^2$

for finding $f(f(x))$: *no marks*

2 in (b) :

for finding $f(g(x))$: *no mark*

for finding $f(f(x))$:

$f(x^2+1)$ *no mark*

$(x^2+1)^2 + 1$ *award* $\bullet^4$

3 There are no marks available for either $g(x) \times f(x)$ or $g(x) \times g(x)$.

1.04

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.04		4	A18	CN	7099	1	1	2	4		

Find the range of values of k such that the equation
$kx^2 - x - 1 = 0$ has no real roots. **4**

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This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	know to use discriminant < 0
$\bullet^2$	ic	interpret the values of a, b and c
$\bullet^3$	ic	substitute
$\bullet^4$	pd	solve an inequation

Primary Method : Give 1 mark for each •

$\bullet^1$ $b^2 - 4ac < 0$
$\bullet^2$ $a = k,\ b = -1,\ c = -1$ s/i by $\bullet^3$
$\bullet^3$ $1 + 4k$
$\bullet^4$ $k < -\frac{1}{4}$

Notes

1 The " < 0 " has to appear at least once at the $\bullet^1$ stage or the $\bullet^3$ stage for $\bullet^1$ to be awarded
2 If an x appears at $\bullet^2$ stage, none of $\bullet^2$, $\bullet^3$ or $\bullet^4$ are available
3 Some candidates may start with the quadratic formula. Apply the marking scheme to the part underneath the square root sign
4 The use of any expression masquerading as the discriminant can only gain $\bullet^2$ at most

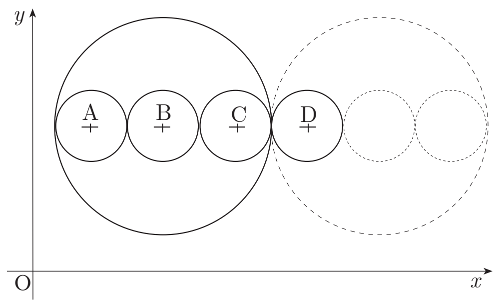
Common Error 1

$\bullet^1$ X $b^2 - 4ac$
$\bullet^2 \checkmark$,$\bullet^3 \checkmark$ $1 + 4k$
$k = -\frac{1}{4}$
$\bullet^4$ X $k < -\frac{1}{4}$

1.05

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.05		5	G10	CN	7041	1	1	3	5		

The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.
Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.
This pattern is continued, as shown in the diagram.
Find the equation of the circle with centre D. **5**



The primary method m.s is based on the following generic m.s.
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$\bullet^1$	ic	state centre
$\bullet^2$	ss	know to, and find, large radius
$\bullet^3$	ss	know to, and find the small radius
$\bullet^4$	ic	interpret new centre
$\bullet^5$	ic	state equation of circle

Primary Method : Give 1 mark for each •

$\bullet^1$	$B = (7,8)$	
$\bullet^2$	$r_{large} = \sqrt{7^2 + 8^2 - 77} = 6$	
$\bullet^3$	$r_{small} = \frac{6}{3}$	s/i by $\bullet^5$
$\bullet^4$	$D = (15,8)$	s/i by $\bullet^5$
$\bullet^5$	$(x-15)^2 + (y-8)^2 = 2^2$	

Note

1 If D = (31,8) then $\bullet^4$ is not available;
however either of
$(x-31)^2 + (y-8)^2 = 2^2$
or $(x-31)^2 + (y-8)^2 = 6^2$
may be awarded $\bullet^5$

2 $\bullet^5$ is only awarded for substituting numerical values for the centre and the radius

1.06

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.06		4	T7	NC	7100	1	2	1	4		

Solve the equation $\sin(2x°) = 6\cos(x°)$ for $0 \le x \le 360$. **4**

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but only where a candidate does not use the primary method or any
alternative method shown in detail in the marking scheme.

- $\bullet^1$ ss know and use double angle formula
- $\bullet^2$ pd write in st. form and factorise
- $\bullet^3$ pd start to solve
- $\bullet^4$ ic know and use exact values

Primary Method : Give 1 mark for each •

- $\bullet^1$ $2\sin(x°)\cos(x°)$
- $\bullet^2$ $\cos(x°)\big(2\sin(x°)-6\big)=0$
- $\bullet^3$ $\cos(x°)=0$ *and* $x=90, 270$
- $\bullet^4$ $\sin(x°)=3$ *and no solution*

or

- $\bullet^3$ $\cos(x°)=0$ *and* $\sin(x°)=3$
- $\bullet^4$ $x=90, 270$ *and no solution*

Notes

1. $\bullet^1$ is NOT available for $2\sin A\cos A$ with no further working
2. The " $=0$ " has to appear at least once at the $\bullet^1$ stage or the $\bullet^2$ stage
3. The inclusion of extra answers which would have been correct but are outside the given interval should be treated as bad form (i.e. not penalised)
4. In following through from an error, $\bullet^4$ is only available for solving an equation with no solution
5. The phrase "no solution" does not always appear after $\sin(x)=3$. The minimum indication that no solution exists might simply be a line drawn through or underneath the equation.

Alt. method : Division by cos(x°)

- $\bullet^1$ $2\sin(x°)\cos(x°)$
- $\bullet^2$ *either* $\cos(x°)=0$ *or* $\cos(x°)\neq 0$ ***stated explicitly***
- $\bullet^3$ $\cos(x°)=0 \Rightarrow x=90$ *or* 270
- $\bullet^4$ $2\sin(x°)=6 \Rightarrow$ *no solution*

1.07

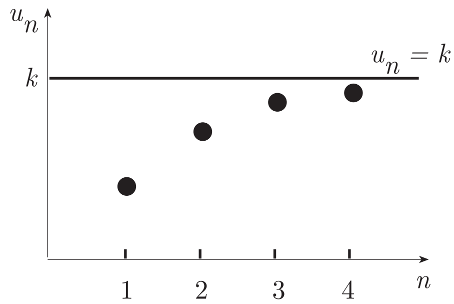
qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.07	a	3	A14	CN	7080		2	1	3		
	b	3				1	1	1	3		

A sequence is defined by the recurrence relation

$$u_{n+1} = \tfrac{1}{4}u_n + 16,\ u_0 = 0.$$

(a) Calculate the values of u_1, u_2 and u_3. **3**

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.
As $n \to \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(b) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k. **3**

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- $\bullet^1$ ic interpret r.r.
- $\bullet^2$ pd process
- $\bullet^3$ pd interpret and process
- $\bullet^4$ ic interpret "a"
- $\bullet^5$ ss know how to find limit
- $\bullet^6$ pd complete

Primary Method : Give 1 mark for each •

- $\bullet^1$ $u_1 = \frac{1}{4}u_0 + 16$ s/i by $\bullet^2$
- $\bullet^2$ 16
- $\bullet^3$ $20, 21$
- $\bullet^4$ $-1 < \frac{1}{4} < 1$
- $\bullet^5$ $k = \frac{1}{4}k + 16$
- $\bullet^6$ $k = \dfrac{64}{3}$

Alternative for $\bullet^5$ and $\bullet^6$

- $\bullet^5$ $k = \dfrac{16}{1 - 0.25}$
- $\bullet^6$ $k = \dfrac{64}{3}$

Notes 1

1 In (a) only numerical values for u_1, u_2 and u_3 are acceptable

2 For (b)(i) accept

$|\frac{1}{4}| < 1$

$0 < \frac{1}{4} < 1$

$\frac{1}{4}$ lies between -1 and 1

$\frac{1}{4}$ is a proper fraction

3 For (b)(i) do ***NOT*** accept

$-1 \le \frac{1}{4} \le 1$

$\frac{1}{4} < 1$

$-1 < a < 1$ unless a is clearly identified/replaced by a $\frac{1}{4}$ anywhere in the answer

Notes 2

4 For (b)(ii)

$k = \dfrac{b}{1-a}$ and nothing else gains no marks

5 For (b)(ii)

$k = \dfrac{16}{\frac{3}{4}}$ *or* $k = \dfrac{16}{0.75}$ may be awarded $\bullet^5$

$k = \dfrac{16}{\frac{3}{4}}$ *or* $k = \dfrac{16}{0.75}$ *or* 21.3 does NOT gain $\bullet^6$

6 Accept L in lieu of k

7 An answer of $\frac{64}{3}$ without any working cannot gain $\bullet^5$ or $\bullet^6$

8 Any calculations based on formulae masquerading as a limit rule cannot gain $\bullet^5$ or $\bullet^6$.

1.08

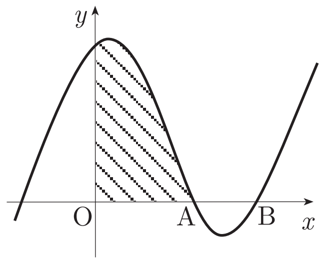
qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.08	a	1	A21, C16	NC	7026	1			1		
	b	3				1	1	1	3		
	c	5				1	2	2	4	1	

The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

(a) Show that the graph cuts the x-axis at (3,0) **1**

(b) Hence or otherwise find the coordinates of A. **3**

(c) Find the shaded area. **5**



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$\bullet^1$	ss	know to evaluate, and evaluate at $x = 3$
$\bullet^2$	ss	strategy for finding other factors
$\bullet^3$	ic	quadratic factor
$\bullet^4$	pd	find +ve root and identify
$\bullet^5$	ss	know to integrate
$\bullet^6$	ic	identify limits
$\bullet^7$	pd	integrate
$\bullet^8$	ic	substitute limits
$\bullet^9$	pd	process limits

Primary Method : Give 1 mark for each •

$\bullet^1$ $'f(3)' = 27 - 36 + 3 + 6 = 0$

$\bullet^2$ $(x-3)(x^2 \; \ldots.)$

$\bullet^3$ $(x-3)(x^2 - x - 2)$

$\bullet^4$ $(x-3)(x-2)(x+1)$ *so* $A = (2,0)$

$\bullet^5$ $\int \left(x^3 - 4x^2 + x + 6\right) dx$

$\bullet^6$ $\int_0^2$

$\bullet^7$ $\frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{1}{2}x^2 + 6x$

$\bullet^8$ $\frac{1}{4}\times 2^4 - \frac{4}{3}\times 2^3 + \frac{1}{2}\times 2^2 + 6\times 2$

$\bullet^9$ $\frac{22}{3}$

Notes

1 The working & evidence for (a) may appear in part (b) and vice versa

2 In Alternative Method 1, $\bullet^1$, candidates must show some acknowledgement of the resulting "zero". Although a statement with respect to the "zero" is preferable, accept something as simple as an underlining of the zero

3 In (c) the appearance of $\int_0^2$ may NOT be used as evidence for $\bullet^4$

4 Since the area is totally above the x-axis, $\bullet^9$ is not available for a negative answer irrespective of whether or not the candidate tries to deal with it

5 For information:

$$\int_0^3 = \frac{27}{4}, \quad \int_0^1 = \frac{65}{12}, \quad \int_0^4 = \frac{32}{3}, \quad \int_0^6 = 90$$

6 For candidates who differentiate, or fail to even try to integrate, $\bullet^7$, $\bullet^8$ and $\bullet^9$ are not available

Alt. Method 1 for $\bullet^1$ *to* $\bullet^4$

$\bullet^2$

$$\begin{array}{c|cccc} 3 & 1 & -4 & 1 & 6 \\ & & 3 & -3 & -6 \\ \hline & 1 & -1 & -2 & \underline{\underline{0}} \end{array}$$

$\bullet^1$

$\bullet^3$ $x^2 - x - 2$

$\bullet^4$ $x = 2$, $x = -1$ ***AND*** $x_A = 2$

Alt. Method 2 for $\bullet^1$ *to* $\bullet^4$

$\bullet^1$ $f(3) = \ldots.. = 0$

$\bullet^2$ try $f(n) = \ldots..$ where $n > 0$

$\bullet^3$ $f(2) = \ldots.. = 0$

$\bullet^4$ $x_A = 2$

1.09

qu	ans	mk	code	calc	source	ss	pd	ic	C	B	A	U1	U2	U3
1.09	a	2	A31	NC	7049	1	1		1	1		2		
	b	7				3	3	1	5	2		7		
	c	1						1		1		1		

A function f is defined by the formula $f(x) = 3x - x^3$.

(a) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(b) Find the coordinates of the stationary points of the function and determine their nature. **7**

(c) Sketch the graph of $y = f(x)$. **1**

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$\bullet^1$	ss	know to use, and use $x = 0$ and $y = 0$
$\bullet^2$	pd	process
$\bullet^3$	ss	know to differentiate
$\bullet^4$	pd	differentiate
$\bullet^5$	ss	know to set derivative to zero
$\bullet^6$	pd	solve
$\bullet^7$	pd	find corresponding y's
$\bullet^8$	ss	know to justify, and justify stationary pts
$\bullet^9$	ic	interpret (e.g. nature table)
$\bullet^{10}$	ic	sketch including relevant points

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$ *any two of* $x = 0$, $x = \sqrt{3}$ *and* $x = -\sqrt{3}$

$\bullet^2$ *remaining one*

$\bullet^3$ $f'(x) =$

$\bullet^4$ $3 - 3x^2$

$\bullet^5$ $f'(x) = 0$

$\bullet^6$ $\bullet^7$

	$\bullet^6$	$\bullet^7$
x	1	-1
y	2	-2 s/i by the sketch

$\bullet^8$ $\bullet^9$

	$\bullet^8$			$\bullet^9$		
...	-1	...	...	1	...
f'	$-$	0	$+$	$+$	0	$-$
	minimum			maximum		

$\bullet^{10}$ *sketch(see below)*

Notes 1

1 $\bullet^2$ is only available if $\bullet^1$ has been awarded

2 The " $= 0$ " shown at $\bullet^5$ must appear at least once somewhere in the working between $\bullet^3$ and $\bullet^6$

3 $\bullet^6$ is only available as a consequence of solving $f'(x) = 0$

4 An unsimplified $\sqrt{1}$ should be penalised at the first occurence

5 The evidence for $\bullet^7$ and $\bullet^9$ may not appear until the sketch

6 The nature table must reflect previous working from $\bullet^4$ and $\bullet^6$

7 The minimum requirement for the sketch is a cubic passing through the origin and with turning points annotated

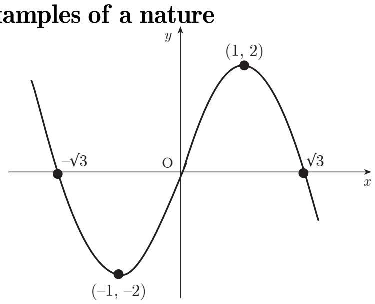
Notes 2

8 The use of the 2nd derivative is an acceptable strategy for $\bullet^8$

9 As shown in the Primary Method, $\bullet^6$ & $\bullet^7$, and $\bullet^8$ & $\bullet^9$ may be marked in series or in parallel [see foot of next page]

10 A " $-\sqrt{3}$ " appearing for the first time on the sketch may not be awarded $\bullet^1$ / $\bullet^2$ retrospectively

11 **See foot of next page for examples of a nature table.**



1.10

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.10		3	C21	CN	7004	2	1			3	

Given that $y = \sqrt{3x^2 + 2}$, find $\dfrac{dy}{dx}$. **3**

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but only where a candidate does not use the primary method or any
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$\bullet^1$ ss expresses in standard form
$\bullet^2$ pd differentiate a binomial to fractional power
$\bullet^3$ ss know and use chain rule

Primary Method : Give 1 mark for each •

$\bullet^1$ $\left(3x^2 + 2\right)^{\frac{1}{2}}$
$\bullet^2$ $\frac{1}{2}\left(3x^2 + 2\right)^{-\frac{1}{2}}$
$\bullet^3$ $\times\, 6x$

see previous page

Marking in series

$\bullet^6$ $x = 1,\ x = -1$
$\bullet^7$ $y = 2,\ y = -2$

Marking in parallel

$\bullet^6$ $x = 1,\ y = 2$
$\bullet^7$ $x = -1,\ y = -2$

Marking in series or parallel

		$\bullet^6$	$\bullet^7$
$\bullet^6$	x	1	-1
$\bullet^7$	y	2	-2

Example of a minimum requirement nature table

		$\bullet^8$			$\bullet^9$	
$\bullet^8$	...	-1	...	...	1	...
f'	$-$	0	$+$	$+$	0	$-$
$\bullet^9$		minimum			maximum	

Example of a preferred nature table

		$\bullet^8$			$\bullet^9$	
x	$\rightarrow$	-1	$\rightarrow$	$\rightarrow$	1	$\rightarrow$
$\bullet^8$ f'	$-$	0	$+$	$+$	0	$-$
	$\ddots$	$\cdots$	$\cdot^{\cdot^{\cdot}}$	$\cdot^{\cdot^{\cdot}}$	$\cdots$	$\ddots$
$\bullet^9$		min at $x = -1$			max at $x = 1$	

Common Errors

1 $\bullet^1$ X $\quad y = (3x^2 + 2)^{-1}$
$\bullet^2$ X $\quad \dfrac{dy}{dx} = -(3x^2 + 2)^{-2}$
$\bullet^3$ X ✓ $\quad ... \times 6x$

2 $\bullet^1$ ✓ $\quad y = (3x^2 + 2)^{\frac{1}{2}}$
$\bullet^2$ X $\quad \dfrac{dy}{dx} = -\dfrac{1}{2}(3x^2 + 2)^{\frac{3}{2}}$
$\bullet^3$ X ✓ $\quad ... \times 6x$

1.11

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
1.11	a	4	T13, T15	NC	7006	1	2	1	4		
	b	4						4		2	2

(a) Express $f(x) = \sqrt{3}\cos(x) + \sin(x)$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(b) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \le x \le 2\pi$. **4**

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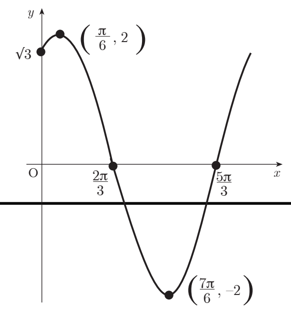
- $\bullet^1$ ss know to use, and use compound formula
- $\bullet^2$ ic equates coefficients
- $\bullet^3$ pd finds k
- $\bullet^4$ pd finds a
- $\bullet^5$ ic interprets a
- $\bullet^6$ ic interprets k
- $\bullet^7$ ic sketch with x-intercepts
- $\bullet^8$ ic sketch with y-intercept

Primary Method : Give 1 mark for each •

- $\bullet^1$ $k\cos(x)\cos(a) + k\sin(x)\sin(a)$ ***stated explicitly***
- $\bullet^2$ $k\cos(a) = \sqrt{3},\ k\sin(a) = 1$ ***stated explicitly***
- $\bullet^3$ $k = 2$
- $\bullet^4$ $a = \frac{\pi}{6}$

a sketch showing

- $\bullet^5$ $\max\left(\frac{\pi}{6},...\right)$ *and* $\min\left(\frac{7\pi}{6},...\right)$
- $\bullet^6$ $\max\left(...,2\right)$ *and* $\min\left(....,-2\right)$
- $\bullet^7$ $\left(\frac{2\pi}{3},0\right)$ *and* $\left(\frac{5\pi}{3},0\right)$
- $\bullet^8$ $\left(0,\sqrt{3}\right)$



Notes 1

1 In the whole question, do not penalise more than once for not using radians
Table showing marks lost for using degrees:

a	$30°$	$\frac{\pi}{6}$	$60°$	$\frac{\pi}{3}$
graph in degrees	**−1**	**−1**	**−2**	**−2**
graph in radians	**−1**	**OK**	**−1**	**−1**

In (a)

2 $k(\cos x \cos a + \sin x \sin a)$ is acceptable for $\bullet^1$

3 $k = \sqrt{4}$ does NOT earn $\bullet^3$

4 $2(\cos x \cos a + \sin x \sin a)$ etc is acceptable for $\bullet^1$ & $\bullet^3$

5 Candidates may use any form of the wave equation as long as their final answer is in the form $k\cos(x - a)$. If not then $\bullet^4$ is not available

6 Treat $k\cos x \cos a + \sin x \sin a$ as bad form ONLY if $\bullet^2$ is gained.

Notes 2

In (b)

7 Do not penalise graphs which go beyond $0 \le x \le 2\pi$

8 A maximum of 3 marks are available for candidates who attempt to sketch graphs of $k\cos(x + a)$, $k\sin(x + a)$ *or* $k\sin(x - a)$. No other graphs can earn any credit

9 **Alternative marking for 2 marks for candidates who do not make a sketch**

$\max\left(\frac{\pi}{6},...\right)$, $\min\left(\frac{7\pi}{6},...\right)$, $(...,2)$, $(...,-2)$, $\left(\frac{2\pi}{3},0\right)$, $\left(\frac{5\pi}{3},0\right)$ *and* $\left(0,\sqrt{3}\right)$

- $\bullet^5$ *any two from the above list*
- $\bullet^6$ *another two from the above list*

2007 Mathematics

Higher – Paper 2

Finalised Marking Instructions

2.01

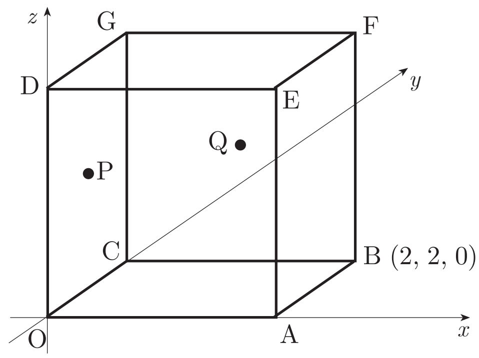
qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
2.01	a	1	G21, G28	CN	7044			1	1		
	b	2		CN				2	2		
	c	5		CN		1	4		5		

OABCDEFG is a cube with side 2 units, as shown in the diagram.
B has coordinates (2, 2, 0).
P is the centre of face OCGD and Q is the centre of face CBFG.

(*a*) Write down the coordinates of G. **1**

(*b*) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. **2**

(*c*) Find the size of angle POQ. **5**



The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ic	interpret 2-D sketch of 3-D situation
$\bullet^2$	ic	interpret coordinates to vector
$\bullet^3$	ic	interpret coordinates to vector
$\bullet^4$	ss	knows to use scalar product
$\bullet^5$	pd	process length
$\bullet^6$	pd	process length
$\bullet^7$	pd	process process scalar product
$\bullet^8$	pd	process angle

Primary Method : Give 1 mark for each •

$\bullet^1$ $G = (0,2,2)$

$\bullet^2$ $\boldsymbol{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$\bullet^3$ $\boldsymbol{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\boldsymbol{p}$ and $\boldsymbol{q}$ must be stated explicitly as a column (or row) vector

$\bullet^4$ $\cos P\hat{O}Q = \dfrac{\boldsymbol{p}.\boldsymbol{q}}{|\boldsymbol{p}||\boldsymbol{q}|}$ stated or implied (s/i) by $\bullet^8$

$\bullet^5$ $|\boldsymbol{p}| = \sqrt{2}$

$\bullet^6$ $|\boldsymbol{q}| = \sqrt{6}$

$\bullet^7$ $\boldsymbol{p}.\boldsymbol{q} = 3$

$\bullet^8$ $P\hat{O}Q = 30°$

$[radians : \frac{\pi}{6}\ (0.524);\ gradians : 33.3\ ]$

Notes 1

1. Treat coordinates written as column vectors as bad form
2. In (b), if $\boldsymbol{p}$ is wrong, this ***may*** be a follow through from (a) which has wrong coordinates for G.
3. For candidates who do not attempt $\bullet^8$, the formula quoted at $\bullet^4$ must relate to the labelling in the question for $\bullet^4$ to be awarded.
4. In (c) for $\bullet^8$ accept answers which round to 30°(2 s.f.)
5. In (c) $\bullet^4$ is not available for candidates who choose to calculate an incorrect angle (e.g. angle OPQ).

Alternative Method for $\bullet^4$ ***to*** $\bullet^8$

$\bullet^4$ $\cos P\hat{O}Q = \dfrac{OP^2 + OQ^2 - PQ^2}{2 \times OP \times OQ}$ stated or implied (s/i) by $\bullet^8$

$\bullet^5$ $OP = \sqrt{2}$

$\bullet^6$ $OQ = \sqrt{6}$

$\bullet^7$ $PQ = \sqrt{2}$

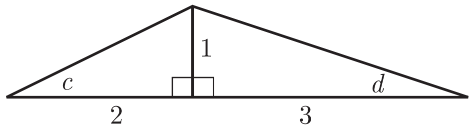
$\bullet^8$ $P\hat{O}Q = 30°$

$[radians : \frac{\pi}{6}\ (0.524);\ gradians : 33.3\ ]$

2.02

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
2.02	a	4	T9	CN	7098	1	1	2	4		
	b	4				2	1	1	4		

The diagram shows two right-angled triangles with angles c and d marked as shown.



(a) Find the exact value of $\sin(c+d)$. **4**

(b) (i) Find the exact value of $\sin 2c$

(ii) Show that $\cos 2d$ has the same exact value. **4**

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ic	interpret the diagram
$\bullet^2$	ss	expand
$\bullet^3$	ic	substitute
$\bullet^4$	pd	simplify
$\bullet^5$	ss	use double angle formula
$\bullet^6$	pd	process
$\bullet^7$	ss	use double angle formula
$\bullet^8$	ic	complete proof of equality

Primary Method : Give 1 mark for each •

$\bullet^1$	$\sqrt{5}$ *and* $\sqrt{10}$	s/i by $\bullet^3$
$\bullet^2$	$\sin(c)\cos(d)+\cos(c)\sin(d)$	s/i by $\bullet^3$
$\bullet^3$	$\frac{1}{\sqrt{5}}\times\frac{3}{\sqrt{10}}+\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}$	
$\bullet^4$	$\frac{1}{\sqrt{2}}$ (accept any equivalent single fraction)	
$\bullet^5$	$2\sin(c)\cos(c)$	
$\bullet^6$	$2\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}=\frac{4}{5}$ *or equivalent*	
$\bullet^7$	*e.g.* $\cos^2(d)-\sin^2(d)$	
$\bullet^8$	$\frac{9}{10}-\frac{1}{10}=\frac{8}{10}=\frac{4}{5}$	

Notes 1

1 Any attempt to use $\sin(c+d)=\sin c+\sin d$ loses $\bullet^2$, $\bullet^3$ and $\bullet^4$

2 At $\bullet^3$ treat $\sin\left(\frac{1}{\sqrt{5}}\right)\cos\left(\frac{3}{\sqrt{10}}\right)+\cos\left(\frac{2}{\sqrt{5}}\right)\sin\left(\frac{1}{\sqrt{10}}\right)$ as bad form if the trig functions disappear to give the answer

3 At the $\bullet^3$ stage do not penalise the use of fractions which are greater than 1

4 Neither $\bullet^4$ nor $\bullet^6$ are available for answers >1

5 Any work based on $\sin 2c = 2\sin c$ loses $\bullet^5$ and $\bullet^6$

6 Any work based on $\cos 2d = 2\cos d$ loses $\bullet^7$ and $\bullet^8$

7 In (b) candidates may calculate $\sin 2c$ and $\cos 2d$ in any order. If either $\sin 2c$ or $\cos 2d$ is correct that may be awarded 2 of the 4 marks available

8 Any working based on numerical values for c and d (eg 27° and 18°) earns no credit but $\bullet^1$, $\bullet^2$, $\bullet^5$ and $\bullet^7$ are still available.

9 $\bullet^8$ is only available if the answer to (b)(ii) is shown to be equivalent to the answer to (b)(i)

10 If $\sqrt{5}$ and $\sqrt{10}$ are approximated to decimal values then $\bullet^4$, $\bullet^6$ and $\bullet^8$ are not available.

Common Errors

1 $\sin 2c = 2\sin d\cos d$

$\sin 2c = 2\frac{1}{\sqrt{10}}\frac{3}{\sqrt{10}}$ award 1 mark from $\bullet^5$ and $\bullet^6$

2 $\cos 2d = \cos^2 c - \sin^2 c$

$\cos 2d = \frac{2}{\sqrt{5}}\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{5}}\frac{1}{\sqrt{5}}$ award 1 mark from $\bullet^7$ and $\bullet^8$

2.03

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
2.03		6	G13	CN		1	1	4	6		

Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	substitute
$\bullet^2$	pd	expand brackets
$\bullet^3$	ic	express in standard form
$\bullet^4$	ic	factorise
$\bullet^5$	ic	complete proof
$\bullet^6$	ic	state coordinates

Primary Method : Give 1 mark for each •

$\bullet^1$ $x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$
$\bullet^2$ $.....36 - 24x + 4x^2 .... - 24 + 8x....$
$\bullet^3$ $5x^2 - 10x + 5 = 0$
$\bullet^4$ $(x-1)^2 = 0$
$\bullet^5$ equal roots $\Rightarrow$ line is tangent
$\bullet^6$ $x = 1, \ \ y = 4$

alternatives for $\bullet^4$ *and* $\bullet^5$

$\bullet^4$ $b^2 - 4ac = 0 \Rightarrow$ tangent
$\bullet^5$ $(-10)^2 - 4 \times 5 \times 5 = 0$

$\bullet^4$ use quad. formula to get roots
$\bullet^5$ equal roots $\Rightarrow$ line is tangent

Alternative Method : Give 1 mark for each •

$\bullet^1$ $m_{line} = -2$
$\bullet^2$ $(-3,2)$ *and* $\frac{1}{2}$
$\bullet^3$ *equ. of radius* $: y - 2 = \frac{1}{2}(x+3)$
$\bullet^4$ $x = 1$
$\bullet^5$ $y = 4$
$\bullet^6$ check that (1,4) lies on the circle

Notes 1

1 An " $= 0$ " must appear somewhere in the working between $\bullet^1$ and $\bullet^4$ stage. Failure to appear will lose one of these marks

2 For candidates who obtain 2 roots:
$\bullet^5$ is still available for "not equal roots so NO tangent" but $\bullet^6$ is not available

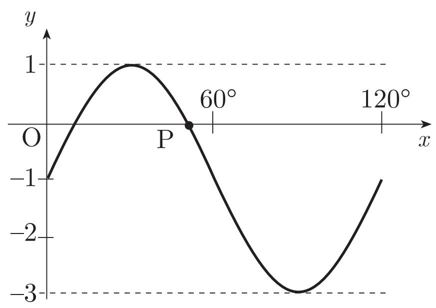
2.04

qu	part	mk	code	calc	source	ss		ic	C	B	A
2.04	a	3	T4, T7	CN	7102			3	3		
	b	3		CN		1	2		3		

The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.

(*a*) Write down the values of a, b and c. **3**

(*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. **3**



The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$ ic interpret vertical scaling
$\bullet^2$ ic interpret period
$\bullet^3$ ic interpret vertical translation
$\bullet^4$ ss set to zero
$\bullet^5$ pd process exact value
$\bullet^6$ ic interpret diagram

solution via a graphics calculator

$\bullet^4$ ss sketch and annotate
$\bullet^5$ ic interpret scale
$\bullet^6$ ic check exact value

Primary Method : Give 1 mark for each •

$\bullet^1$ $a = 2$
$\bullet^2$ $b = 3$
$\bullet^3$ $c = -1$
$\bullet^4$ $2\sin(3x°) - 1 = 0$
$\bullet^5$ one answer from 10° or 50°
$\bullet^6$ $x_P = 50°$

alternative for $\bullet^4$, $\bullet^5$ and $\bullet^6$

$\bullet^4$ sketch of graph with pointer to sol.point
$\bullet^5$ extraction of 50°
$\bullet^6$ confirmation of $2\sin(3 \times 50°) - 1$ does $= 0$

Notes 1

1 $\bullet^4$ may be awarded for $a\sin(bx) + c = 0$
2 For $\bullet^2$ accept " $b = 3x$ " as bad form
3 $\bullet^6$ may only be awarded for a value of x such that $30 < x < 60$
4 $\bullet^6$ may be awarded for (50°,0) but NOT for (0,50°)

2.05

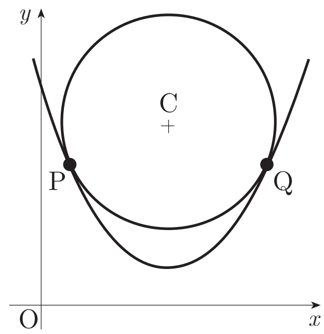
qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
2.05	a	5	C5,G10,G11	CN	7017	2	2	1	5		
	b	2				1		1		2	
	c	2						2		2	

A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.

(a) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

(b) Find the coordinates of P. **2**

(c) Find the coordinates of C, the centre of the circle. **2**



The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	know to differentiate
$\bullet^2$	pd	process
$\bullet^3$	ss	equate gradients
$\bullet^4$	pd	process
$\bullet^5$	ic	interpret y-coordinate
$\bullet^6$	ss	use symmetry of diagram
$\bullet^7$	ic	interpret coordinates
$\bullet^8$	ic	interpret centre
$\bullet^9$	ic	interpret centre

Primary Method : Give 1 mark for each •

$\bullet^1$ $\frac{dy}{dx} = ...$(1 term correct)

$\bullet^2$ $x - 8$

$\bullet^3$ $x - 8 = 4$

$\bullet^4$ $x = 12$

$\bullet^5$ $y = 10$

$\bullet^6$ $m_P = -4$

$\bullet^7$ P = (4,10)

$\bullet^8$ $x_C = 8$

$\bullet^9$ $y_C = 11$

Notes 1

1 Treat $y = x - 8$ as bad form provided y is replaced by 4 at $\bullet^3$

2 *Cave*
Look out for the following:
$\bullet^5$ is not available to candidates who substitute the gradient of 4 into the equation in order to find the value of y_Q

3 Alt. strategies for $\bullet^6$
(a) substitute $y = 10$ into the parabola
(b) use the t.p. as a step to P

4 *Cave*
There are other legitimate methods for finding the coordinates of Q

5 Candidates who solve the tangents at P and Q AND then state that $x_C = 8$ may be awarded $\bullet^8$.

Alternative Method for (c)

Solving the normals

i.e. $y - 10 = -\frac{1}{4}(x - 12)$

$y - 10 = \frac{1}{4}(x - 4)$

may be used. Marks are awarded as normal:

$x = 8$ ($\bullet^8$) and $y = 11$ ($\bullet^9$)

Common Errors

1 $\frac{dy}{dx} = x - 8$ $\quad \surd \bullet^1, \surd \bullet^2$

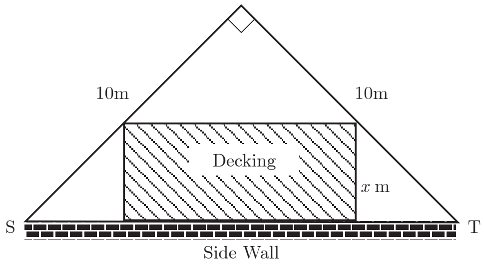
$x - 8 = 0 \Rightarrow x = 8, y = 2$ $\quad \surd \bullet^5$

2 For the occasional candidate who starts with $x - 8 = 4$
award $\bullet^1, \bullet^2$ and $\bullet^3$

2.06

qu	part	mk	code	calc	source	ss		ic	C	B	A
2.06	a	3	C11	CN	7062		1	2			3
	b	5		CN		1	3	1	1	4	

A householder has a garden in the shape of a right-angled isosceles triangle.
It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(a) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2$$ **3**

(b) Find the dimensions of the decking which maximises its area. **5**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ pd calculate ST
- $\bullet^2$ ic interpret the triangle
- $\bullet^3$ ic complete proof
- $\bullet^4$ ss set derivative zero
- $\bullet^5$ pd differentiate
- $\bullet^6$ pd solve for breadth
- $\bullet^7$ ic justify s.p.s with e.g. nature table
- $\bullet^8$ pd find corresponding length

Primary Method : Give 1 mark for each •

- $\bullet^1$ $ST = \sqrt{200}$
- $\bullet^2$ $length = \sqrt{200} - 2x$ s/i by their method
- $\bullet^3$ $\left(\sqrt{200} - 2x\right) \times x$ *and complete proof*
- $\bullet^4$ $\dfrac{dA}{dx} = 0$
- $\bullet^5$ $\dfrac{dA}{dx} = 10\sqrt{2} - 4x$
- $\bullet^6$ $x = \dfrac{10\sqrt{2}}{4}$ or equivalent (3.5)
- $\bullet^7$ *justification* : e.g. nature table
- $\bullet^8$ $length = 5\sqrt{2}$ (7.1)

Notes 1

In (b)

1 An " $= 0$ " must appear somewhere in the working between $\bullet^4$ and $\bullet^6$

2 For $\bullet^7$ accept $\frac{d^2A}{dx^2} = -4 < 0$ at $x = \frac{10\sqrt{2}}{4} \Rightarrow$ maximum

Minimum requirement of a nature table

	...	3.5	...
$f'(x)$	$+$	0	$-$

hence maximum

better would be

x	$\rightarrow$	$\frac{5\sqrt{2}}{2}$	$\rightarrow$
$f'(x)$	$+$	0	$-$
$f(x)$	$\cdot^{\cdot^{\cdot}}$	$\cdots$	$\ddots$

hence maximum

at $x = \frac{5\sqrt{2}}{2}$

2.07

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
2.07		4	C23, T3	CR	7046		3	1		3	1

Find the value of $\int_0^2 \sin(4x+1)\,dx$. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide
but only where a candidate does not use the primary method or any
alternative method shown in detail in the marking scheme.

$\bullet^1$ pd integrate the trig function
$\bullet^2$ pd deal with the "4"
$\bullet^3$ ic substitute the limits
$\bullet^4$ pd evaluate

Primary Method : Give 1 mark for each •

$\bullet^1$ $-\cos(4x+1)$
$\bullet^2$ $\times\dfrac{1}{4}$
$\bullet^3$ $-\dfrac{1}{4}\cos(4\times2+1)-\left(-\dfrac{1}{4}\cos(4\times0+1)\right)$
$\bullet^4$ 0.36

Notes 1

1 $\bullet^2$ is only available if it follows on from $\pm\sin(4x+1)$ *or* $\pm\cos(4x+1)$
2 $\bullet^3$ is available for substituting the limits correctly into any trig. function except the original one
3 $\bullet^4$ is available for using any trig. function except the original one
4 If candidates leave the calculator in degree mode obtaining 0.000304 then $\bullet^4$ is NOT awarded

Alternative Method

$\sin 4x\cos1+\cos4x\sin1$
$\bullet^1$ $-\frac{1}{4}\cos4x\cos1$
$\bullet^2$ $\frac{1}{4}\sin4x\sin1$
$\bullet^3$ $\left(-\frac{1}{4}\cos8\cos1+\frac{1}{4}\sin8\sin1\right)-\left(-\frac{1}{4}\cos0\cos1+\frac{1}{4}\sin0\sin1\right)$
$\bullet^4$ 0.36

2.08

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A
2.08		4	A31	CR	7049	2	1	1		4	

The curve with equation $y = \log_3(x-1) - 2.2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$.
Find the value of a. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ic substitute
- $\bullet^2$ ss isolate the log term
- $\bullet^3$ ss convert to exponential form
- $\bullet^4$ pd process

Primary Method : Give 1 mark for each •

- $\bullet^1$ $\log_3(a-1) - 2.2 = 0$ s/i by $\bullet^2$
- $\bullet^2$ $\log_3(a-1) = 2.2$
- $\bullet^3$ $a - 1 = 3^{2.2}$
- $\bullet^4$ $a = 12.2$

Alt.method 1

- $\bullet^1$ $\log_3(a-1) - 2.2 = 0$ s/i by $\bullet^2$
- $\bullet^2$ $\log_3(a-1) = 2.2$
- $\bullet^3$ $\log_3(a-1) = \log_3(11.21)$
- $\bullet^4$ $a = 12.2$

Alt.method 2

- $\bullet^1$ $\log_3(a-1) - 2.2 = 0$ s/i by $\bullet^2$
  $\log_3(a-1) - 2.2\log_3 3 = 0$
- $\bullet^2$ $\log_3(a-1) - \log_3(11.21) = 0$
- $\bullet^3$ $\log_3 \frac{(a-1)}{11.21} = 0$
- $\bullet^4$ $a = 12.2$

Notes 1

1 Solutions given in terms of x rather than a should be treated as bad form.

Common Error 1

- $\bullet^1$ $\surd$ $\log_3(a-1) - 2.2 = 0$
- $\bullet^2$ $\surd$ $\log_3(a-1) = 2.2$
- $\bullet^3$ X $\log_3(a-1) = \log_3 2.2$
- $\bullet^4$ X $a - 1 = 2.2 \Rightarrow a = 3.2$ [*eased*]

Common Error 2

- $\bullet^1$ $\surd$ $\log_3(a-1) - 2.2 = 0$
- $\bullet^2$ $\surd$ $\log_3(a-1) = 2.2$
- $\bullet^3$ X $\log_3 a - \log_3 1 = 2.2$
  $\log_3 a = 2.2$
- $\bullet^4$ $X\surd$ $a = 3^{2.2} = 11.2$

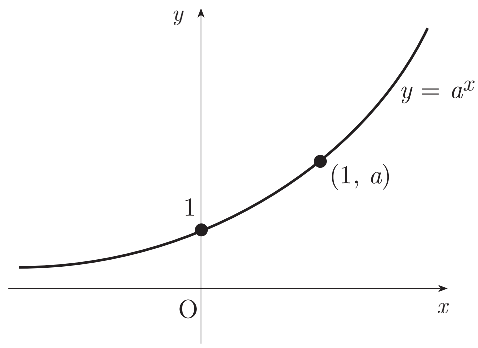
2.09

qu	part	mk	code	calc	source	ss		ic	C	B	A		U1	U2	U3
2.09	a	2	A3	CN	7071			2		2			2		
	b	2		CN				2			2		2		

The diagram shows the graph of $y = a^x$, $a > 1$.
On separate diagrams sketch the graphs of:

(a) $y = a^{-x}$ **2**

(b) $y = a^{1-x}$ **2**

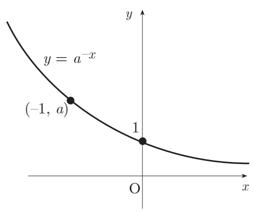


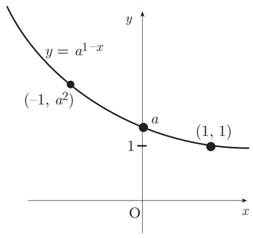
The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$ ic determine the requ. transformation
$\bullet^2$ ic state coordinates of pt. on graph
$\bullet^3$ ic determine the requ. transformation
$\bullet^4$ ic state coordinates of pt. on graph

Primary Method : Give 1 mark for each •

$\bullet^1$ reflecting in y-*axis* and passing thr' e.g. (0,1)
$\bullet^2$ passing thr' 1 more point e.g. $(-1, a)$ *or* $\left(1, \frac{1}{a}\right)$
$\bullet^3$ vertical scaling of "a" and passing thr' e.g. $(0, a)$
$\bullet^4$ passing thr' 1 more point e.g. $(-1, a^2)$ *or* (1,1)





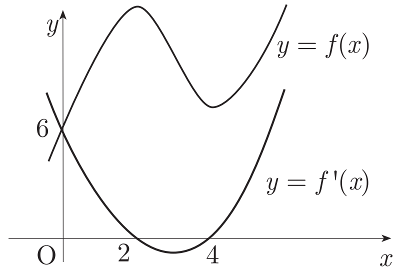
Notes 1

1. For $\bullet^1$ and $\bullet^3$ the shape must be an exponential decay graph lying above the x-axis
2. There are no follow-through marks available to candidates who use an incorrect graph from (a) as a basis for their answer to (b).

2.10

qu	part	mk	code	calc	source	ss		ic	C	B	A
2.10	a	3	C18, C19	CN	7028	1	1	1	1	2	
	b	4		CN		1	1	2			4

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.
Both graphs pass through the point (0,6).
The graph of $y = f'(x)$ also passes through the points (2,0) and (4,0).



(a) Given that $f'(x)$ is of the form $k(x-a)(x-b)$

(i) Write down the values of a and b.

(ii) Find the value of k. **3**

(b) Find the equation of the graph of the cubic function $y = f(x)$. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ic	interpret roots on diagram
$\bullet^2$	ss	know to use y-intercept
$\bullet^3$	pd	process
$\bullet^4$	ss	know to integrate
$\bullet^5$	pd	integrate
$\bullet^6$	ic	express as an equation
$\bullet^7$	ic	interpret constant of integration

Primary Method : Give 1 mark for each •

$\bullet^1$ $a = 2$ *and* $b = 4$ ***or*** $k(x-2)(x-4)$

$\bullet^2$ $6 = k(0-2)(0-4)$

$\bullet^3$ $k = \dfrac{3}{4}$

$\bullet^4$ $\int \left(\frac{3}{4}(x-2)(x-4)\right) dx$ s/i by $\bullet^5$

$\bullet^5$ *any two terms integrated correctly* ($\frac{3}{12}x^3$ *etc*)

$\bullet^6$ $y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + c$

$\bullet^7$ $c = 6$

Notes 1

1 For candidates who fail to complete (a) but produce values for k, a and b *ex nihilo*, all 4 marks are available in (b).
A deduction of 1 mark may be made if their choice eases the working.

2 In (b)
For candidates who use $k = 1$, a "fully correct" follow-through solution may be awarded 3 out of the last 4 marks

3 For candidates who retain "k", "a" and "b", $\bullet^4$,$\bullet^5$,$\bullet^6$ and $\bullet^7$ are still available.

2.11

qu	part	mk	code	calc	source	ss	pd	ic	C	B	A	U1	U2	U3
2.11	a	1	A33	CR	7014			1		1				1
	b	1						1	1					1
	c	4				1		3			4			4

Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(a) Find the value of a if $(a, 6)$ lies on the graph with equation $y = 3 \times 4^x$. **1**

(b) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

(c) A graph is drawn of $\log_{10} y$ against x. Show that its equation will be of the form $\log_{10} y = Px + Q$ and state the gradient of this line. **4**

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- $\bullet^1$ ic interprets equation
- $\bullet^2$ ic interprets equation
- $\bullet^3$ ss introduces logs
- $\bullet^4$ ic uses log law
- $\bullet^5$ ic uses log law and completes
- $\bullet^6$ ic interprets equation

Primary Method : Give 1 mark for each •

- $\bullet^1$ $a = \dfrac{1}{2}$
- $\bullet^2$ $b = \dfrac{3}{2}$
- $\bullet^3$ $\log_{10}(y) = \log_{10}\left(3 \times 4^x\right)$
- $\bullet^4$ $\log_{10}(y) = \log_{10}(3) + \log_{10}\left(4^x\right)$
- $\bullet^5$ $\log_{10}(y) = x\log_{10}(4) + \log_{10}(3)$
- $\bullet^6$ $gradient = \log_{10}(4)$ or equivalent

Notes

1 Do not penalise $x = \dfrac{1}{2}$, $y = \dfrac{3}{2}$

2 Candidates who start their "proof" with the wrong form (e.g. $y = Px^Q$) earn no credit in part (c).

Alternative Method

- $\bullet^1$ $y = 10^{Px+Q}$
- $\bullet^2$ $y = 10^Q \times (10^P)^x$
- $\bullet^3$ $10^Q = 3 \ and \ 10^P = 4$
- $\bullet^4$ $P = \log_{10} 4$

Cave

In (a) look out for the following :

$$6 = 3 \times 4^a$$
$$2 = 4^a$$
$$\frac{2}{4} = a$$
$$a = \frac{1}{2}$$

This is not awarded $\bullet^1$

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or **X**✓). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pd mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

$\wedge$ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

		margins	
$\frac{dy}{dx} = 4x - 7$	✓ •		
$4x - 7 = \underline{0}$	✗		
$x = \frac{7}{4}$			
$y = 3\frac{7}{8}$	•		2
$C = \underline{(1,-1)}$	✗		
$m = \dfrac{3-(-1)}{4-1}$			
$m_{rad} = \frac{4}{3}$	•		
$m_{tgt} = \dfrac{-1}{\frac{4}{3}}$			
$m_{tgt} = -\frac{3}{4}$	•		
$y - 3 = -\frac{3}{4}(x-2)$	•		3
$x^2 - 3x = 28$	✓ •		
$x = 7$ $\wedge$			1
$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$	✓ •		1

Remember - No comments on the scripts. No abreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

1	2		*UNIT 1*
		A1	*determine range/domain*
		A2	*recognise general features of graphs:poly,exp,log*
		A3	*sketch and annotate related functions*
		A4	*obtain a formula for composite function*
		A5	*complete the square*
		A6	*interpret equations and expressions*
		A7	*determine function(poly,exp,log) from graph & vv*
		A8	*sketch/annotate graph given critical features*
		A9	*interpret loci such as st.lines,para,poly,circle*
		A10	*use the notation* u_n *for the nth term*
		A11	*evaluate successive terms of a RR*
		A12	*decide when RR has limit/interpret limit*
		A13	*evaluate limit*
		A14	*apply A10-A14 to problems*
		G1	*use the distance formula*
		G2	*find gradient from 2 pts,/angle/equ. of line*
		G3	*find equation of a line*
		G4	*interpret all equations of a line*
		G5	*use property of perpendicular lines*
		G6	*calculate mid-point*
		G7	*find equation of median, altitude,perp. bisector*
		G8	*apply G1-G7 to problems eg intersect.,concur.,collin.*
		C1	*differentiate sums, differences*
		C2	*differentiate negative & fractional powers*
		C3	*express in differentiable form and differentiate*
		C4	*find gradient at point on curve & vv*
		C5	*find equation of tangent to a polynomial/trig curve*
		C6	*find rate of change*
		C7	*find when curve strictly increasing etc*
		C8	*find stationary points/values*
		C9	*determinenature of stationary points*
		C10	*sketch curvegiven the equation*
		C11	*apply C1-C10 to problems eg optimise, greatest/least*
		T1	*use gen. features of graphs of* $f(x)=k\sin(ax+b)$,
			$f(x)=k\cos(ax+b)$; *identify period/amplitude*
		T2	*use radians inc conversion from degrees & vv*
		T3	*know and use exact values*
		T4	*recognise form of trig. function from graph*
		T5	*interpret trig. equations and expressions*
		T6	*apply T1-T5 to problems*

1	2		*UNIT 2*
		A15	*use the general equation of a parabola*
		A16	*solve a quadratic inequality*
		A17	*find nature of roots of a quadratic*
		A18	*given nature of roots, find a condition on coeffs*
		A19	*form an equation with given roots*
		A20	*apply A15-A19 to solve problems*
		A21	*use Rem Th. For values, factors, roots*
		A22	*solve cubic and quartic equations*
		A23	*find intersection of line and polynomial*
		A24	*find if line is tangent to polynomial*
		A25	*find intersection of two polynomials*
		A26	*confiirm and improve on approx roots*
		A27	*apply A21-A26 to problems*
		G9	*find C/R of a circle from its equation/other data*
		G10	*find the equation of a circle*
		G11	*find equation of a tangent to a circle*
		G12	*find intersection of line & circle*
		G13	*find if/when line is tangent to circle*
		G14	*find if two circles touch*
		G15	*apply G9-G14 to problems*
		C12	*find integrals of* px^n *and sums/diffs*
		C13	*integrate with negative & fractional powers*
		C14	*express in integrable form and integrate*
		C15	*evaluate definite integrals*
		C16	*find area between curve and x-axis*
		C17	*find area between two curves*
		C18	*solve differential equations(variables separable)*
		C19	*apply C12-C18 to problems*
		T7	*solve linear & quadratic equations in radians*
		T8	*apply compound and double angle (c & da) formulae*
			in numerical & literal cases
		T9	*apply c & da formulae in geometrical cases*
		T10	*use c & da formulaewhen solving equations*
		T11	*apply T7-T10 to problems*

1	2		*UNIT 3*
		A28	*use the laws of logs to simplify/find equiv. expression*
		A29	*sketch associated graphs*
		A30	*solve equs of the form* $A = Be^{kt}$ *for A,B,k or t*
		A31	*solve equs of the form* $\log_b(a) = c$ *for a,b or c*
		A32	*solve equations involving logarithms*
		A33	*use relationships of the form* $y = ax^n$ *or* $y = ab^x$
		A34	*apply A28-A33 to problems*
		G16	*calculate the length of a vector*
		G17	*calculate the 3rd given two from A,B and vector AB*
		G18	*use unit vectors*
		G19	*use: if* $\boldsymbol{u}, \boldsymbol{v}$ *are parallel then* $\boldsymbol{v} = k\boldsymbol{u}$
		G20	*add, subtract, find scalar mult. of vectors*
		G21	*simplify vector pathways*
		G22	*interpret 2D sketches of 3D situations*
		G23	*find if 3 points in space are collinear*
		G24	*find ratio which one point divides two others*
		G25	*given a ratio, find/interpret 3rd point/vector*
		G26	*calculate the scalar product*
		G27	*use: if* $\boldsymbol{u}, \boldsymbol{v}$ *are perpendicular then* $\boldsymbol{v}.\boldsymbol{u}=\mathbf{0}$
		G28	*calculate the angle between two vectors*
		G29	*use the distributive law*
		G30	*apply G16-G29 to problems eg geometry probs.*
		C20	*differentiate* $p\sin(ax+b)$, $p\cos(ax+b)$
		C21	*differentiate using the chain rule*
		C22	*integrate* $(ax + b)^n$
		C23	*integrate* $p\sin(ax+b)$, $p\cos(ax+b)$
		C24	*apply C20-C23 to problems*
		T12	*solve sim.equs of form* $k\cos(a)=p$, $k\sin(a)=q$
		T13	*express* $p\cos(x)+q\sin(x)$ *in form* $k\cos(x\pm a)$ *etc*
		T14	*find max/min/zeros of* $p\cos(x)+q\sin(x)$
		T15	*sketch graph of* $y=p\cos(x)+q\sin(x)$
		T16	*solve equ of the form* $y=p\cos(rx)+q\sin(rx)$
		T17	*apply T12-T16 to problems*