W100/301

NATIONAL	FRIDAY, 18 JANUARY	MATHEMATICS
QUALIFICATIONS	9.00 AM – 10.10 AM	HIGHER
2002		Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

1   Calculators may **NOT** be used in this paper.

2   Full credit will be given only where the solution contains appropriate working.

3   Answers obtained by readings from scale drawings will not receive any credit.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a|\,|b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
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Table of standard integrals:

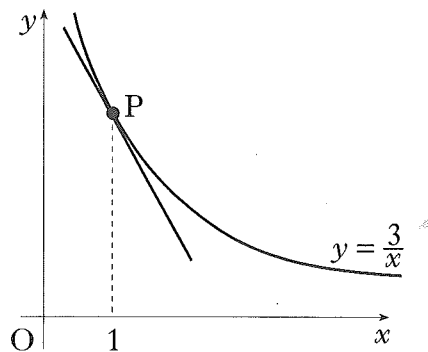| $f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. (a) Find the equation of the straight line through the points A(−1, 5) and B(3, 1).   2

    (b) Find the size of the angle which AB makes with the positive direction of the x-axis.   2

2. (a) If $u = \begin{pmatrix} 1 \\ 7 \\ -2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, write down the components of $u + 3v$ and $u − 3v$.   2

    (b) Hence, or otherwise, show that $u + 3v$ and $u − 3v$ are perpendicular.   2

3. Find the equation of the tangent to the curve with equation $y = \frac{3}{x}$ at the point P where $x = 1$.   5


4. (a) Write down the exact values of $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$.   1

    (b) If $\tan x = 4\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)$, find the exact values of x for $0 \le x \le 2\pi$.   2

5. Given that $(x − 2)$ and $(x + 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d.   5

[Turn over

Marks

6. The side view of part of a roller coaster ride is shown by the path PQRS.
   The curve PQ is an arc of the circle with equation $x^2 + y^2 + 4x - 10y + 9 = 0$.
   The curve QRS is part of the parabola with equation $y = -x^2 + 6x - 5$.
   The point Q has coordinates (2, 3).
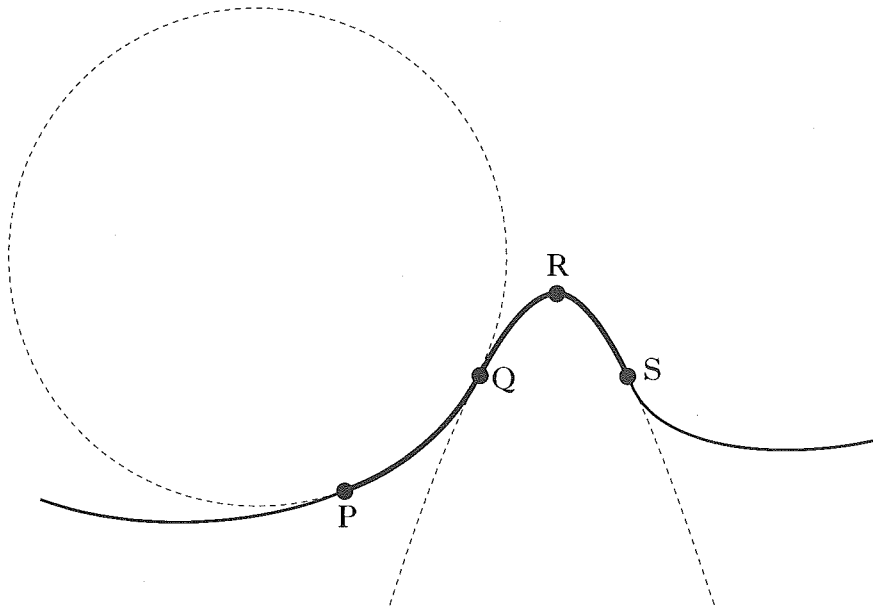
    (a) Find the equation of the tangent to the circle at Q.   **4**

    (b) Show that this tangent to the circle at Q is also the tangent to the parabola
   at Q.   **2**

7. Find $\int \left(\sqrt[3]{x} - \dfrac{1}{\sqrt{x}} \right) dx$.   **4**

8. The diagram shows part of the graph
   of $y = 2^x$.

    (a) Sketch the graph of $y = 2^{-x} - 8$.   **2**

    (b) Find the coordinates of the points
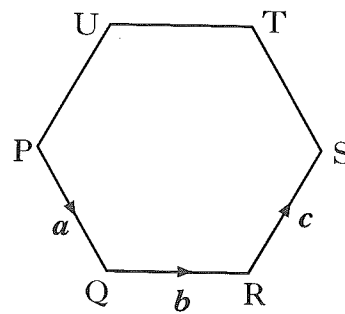   where it crosses the x and y axes.   **2**
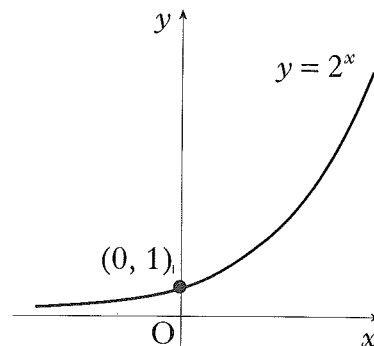
Marks

9. The function f, defined on a suitable domain, is given by $f(x) = \dfrac{3}{x+1}$.

   (a) Find an expression for $h(x)$ where $h(x) = f(f(x))$, giving your answer as a fraction in its simplest form.   **3**

   (b) Describe any restriction on the domain of h.   **1**

10. A function f is defined by $f(x) = 2x + 3 + \dfrac{18}{x-4}$, $x \neq 4$.

    Find the values of x for which the function is increasing.   **5**

11. PQRSTU is a regular hexagon of side 2 units.
    $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{RS}$ represent vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively.

    Find the value of $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c})$.   **3**

12. If $\log_a p = \cos^2 x$ and $\log_a r = \sin^2 x$, show that $pr = a$.   **3**

[END OF QUESTION PAPER]

W100/303

NATIONAL QUALIFICATIONS 2002	FRIDAY, 18 JANUARY 10.30 AM – 12.00 NOON	MATHEMATICS HIGHER Units 1, 2 and 3 Paper 2

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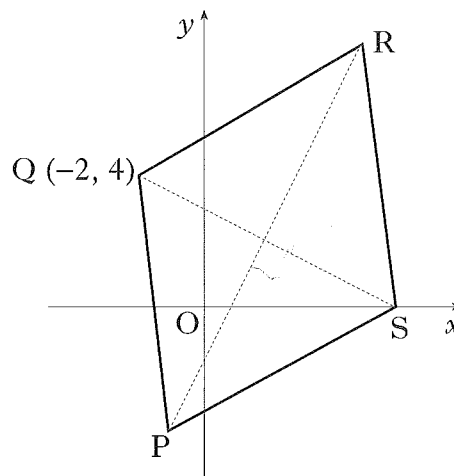1. The diagram shows a rhombus PQRS with its diagonals PR and QS.

   PR has equation $y = 2x - 2$.

   Q has coordinates $(-2, 4)$.

   (a) (i) Find the equation of the diagonal QS.

   (ii) Find the coordinates of T, the point of intersection of PR and QS.

   6

   (b) R is the point $(5, 8)$. Write down the coordinates of P.

   2

2. With reference to a suitable set of coordinate axes, A, B and C are the points $(-8, 10, 20)$, $(-2, 1, 8)$ and $(0, -2, 4)$ respectively.

   Show that A, B and C are collinear and find the ratio AB : BC.

   4

3. (a) Calculate the limit as n $\rightarrow \infty$ of the sequence defined by $u_{n+1} = 0 \cdot 9u_n + 10$, $u_0 = 1$.

   3

   (b) Determine the least value of n for which u_n is greater than half of this limit and the corresponding value of u_n.

   2

4. (a) Write $\sqrt{3} \sin x° + \cos x°$ in the form $k \sin(x + a)°$ where $k > 0$ and $0 \le a < 360$.

   4

   (b) Hence find the maximum value of $5 + \sqrt{3} \sin x° + \cos x°$ and determine the corresponding value of x in the interval $0 \le x \le 360$.
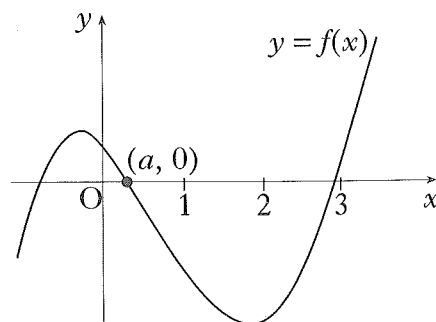
   2

5. Solve the equation $\cos 2x - 2\sin^2 x = 0$ in the interval $0 \le x < 2\pi$.

   4

[Turn over

Marks

6. The graph of $f(x) = 2x^3 - 5x^2 - 3x + 1$ has been sketched in the diagram shown.

   Find the value of a correct to one decimal place.
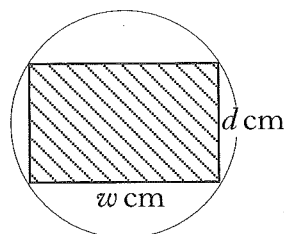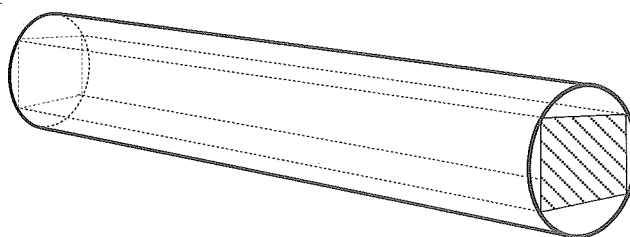
   3

7. A rectangular beam is to be cut from a cylindrical log of diameter 20 cm.

   The diagram shows a cross-section of the log and beam where the beam has a breadth of w cm and a depth of d cm.

   The strength S of the beam is given by

   $S = 1 \cdot 7\, w\, (400 - w^2).$

   Find the dimensions of the beam for maximum strength.

   5

8. Find $\displaystyle\int_0^1 \left(\cos(3x) - \sin\left(\tfrac{1}{3}x + 1\right)\right) dx$ correct to 3 decimal places.

   3

9. A researcher modelled the size N of a colony of bacteria t hours after the beginning of her observations by $N(t) = 950 \times (2 \cdot 6)^{0 \cdot 2t}$.

   (a) What was the size of the colony when observations began?

   1

   (b) How long does it take for the size of the colony to be multiplied by 10?

   4

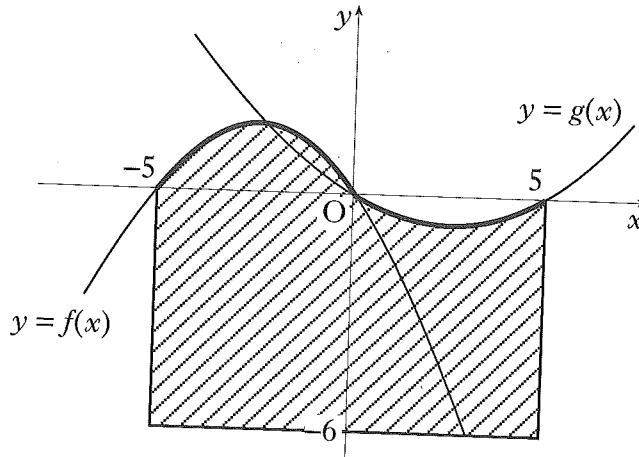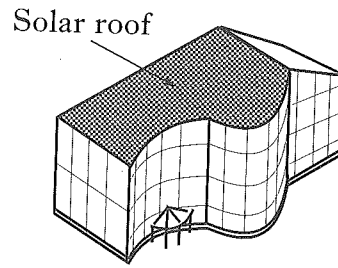10. The line $y + 2x = k$, $k > 0$, is a tangent to the circle $x^2 + y^2 - 2x - 4 = 0$.

    (a) Find the value of k.

    7

    (b) Deduce the coordinates of the point of contact.

    2

Marks

11. An energy efficient building is designed with solar cells covering the whole of its south facing roof. The energy generated by the solar cells is directly proportional to the area, in square units, of the solar roof.

Solar roof

The shape of the solar roof can be represented on the coordinate plane as the shaded area bounded by the functions $f(x) = \frac{1}{4}\left(-x^2 - 5x\right)$, $g(x) = \frac{1}{12}\left(x^2 - 5x\right)$ and the lines $x = -5$, $x = 5$ and $y = -6$.

(a) Find the area of the solar roof.

7

(b) Ten square units of solar cells generate a maximum of 1 kilowatt.

What is the maximum energy the solar roof can generate in kilowatts (to the nearest kilowatt)?

1

[END OF QUESTION PAPER]

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