

X100/301

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
9.00 AM - 10.10 AM

**MATHEMATICS
HIGHER**

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

1. Find the equation of the line which passes through the point $(-1, 3)$ and is perpendicular to the line with equation $4x + y - 1 = 0$. 3

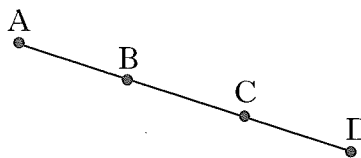
2. (a) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. 2
 (b) Hence or otherwise sketch the graph of $y = f(x)$. 2

3. Vectors \mathbf{u} and \mathbf{v} are defined by $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$.
 Determine whether or not \mathbf{u} and \mathbf{v} are perpendicular to each other. 2

4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.
 (a) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q . 2
 (b) Find the limit of this recurrence relation as $n \rightarrow \infty$. 2

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. 5

6. A and B are the points $(-1, -3, 2)$ and $(2, -1, 1)$ respectively.
 B and C are the points of trisection of AD, that is $AB = BC = CD$.
 Find the coordinates of D. 3



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. 5

8. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. 4

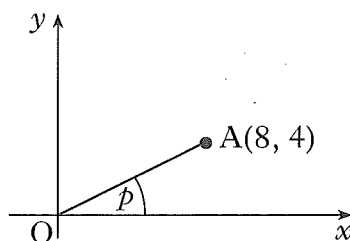
9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.
 (a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. 2
 (b) Write down any restriction on the domain of h . 1

[Turn over for Questions 10 to 12 on Page four

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x -axis.

(a) Find the exact values of:

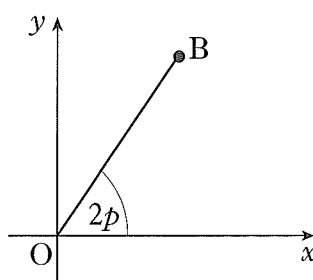
- (i) $\sin(2p)$;
 (ii) $\cos(2p)$.



5

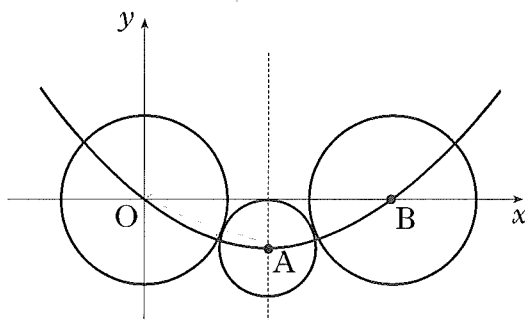
The line OB is inclined at an angle $2p$ radians to the x -axis.

(b) Write down the exact value of the gradient of OB.



1

11. • O, A and B are the centres of the three circles shown in the diagram below.
 • The two outer circles are congruent and each touches the smallest circle.
 • Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
 • The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(a) (i) State the coordinates of A and find the length of the line OA. 2

(ii) Hence find the equation of the circle with centre B. 3

(b) The equation of the parabola can be written in the form $y = px(x + q)$.

Find the values of p and q . 2

12. Simplify $3 \log_e(2e) - 2 \log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. 4

[END OF QUESTION PAPER]

X100/303

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
10.30 AM – 12.00 NOON

**MATHEMATICS
HIGHER**
Units 1, 2 and 3
Paper 2

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Marks

1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

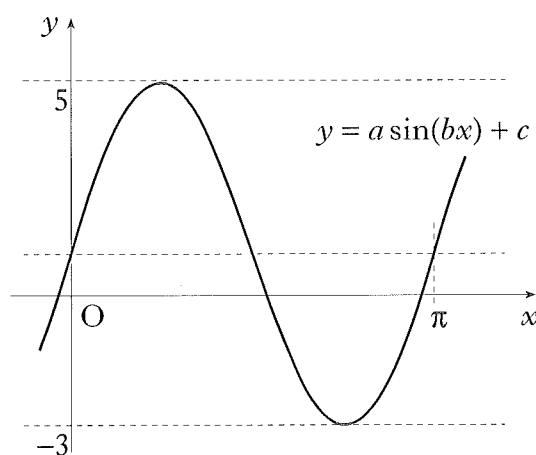
(a) Show that $(x - 2)$ is a factor of $f(x)$.

(b) Express $f(x)$ in its fully factorised form.

4

2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a \sin(bx) + c$.

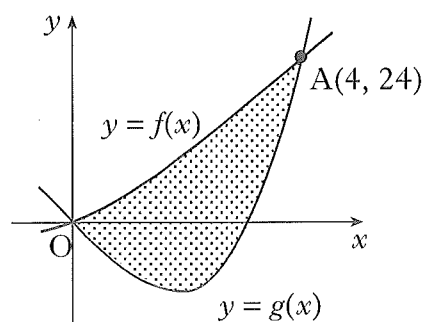
Determine the values of a , b and c .



3

3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at $A(4, 24)$ and the origin.

Find the shaded area enclosed between the curves.



5

4. (a) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$.

5

(b) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact.

6

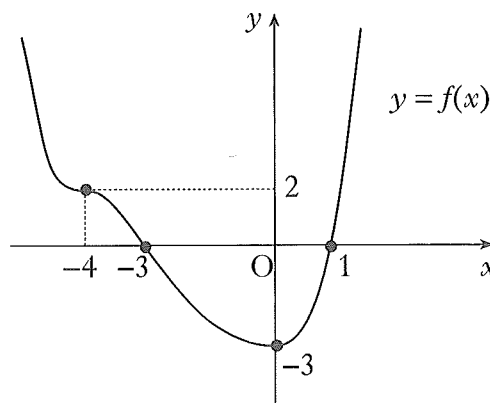
[Turn over

Marks

5. The diagram shows the graph of a function f .

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

- (a) Sketch the graph of $y = f(-x)$.
 (b) On the same diagram, sketch the graph of $y = 2f(-x)$.



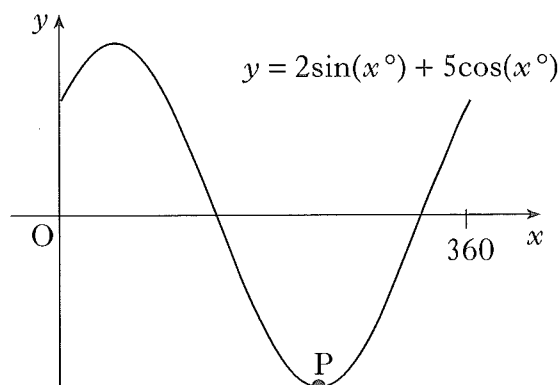
2
2

6. If $f(x) = \cos(2x) - 3 \sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$.

4

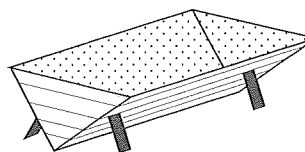
7. Part of the graph of $y = 2\sin(x^\circ) + 5\cos(x^\circ)$ is shown in the diagram.

- (a) Express $y = 2\sin(x^\circ) + 5\cos(x^\circ)$ in the form $k\sin(x^\circ + a^\circ)$ where $k > 0$ and $0 \leq a < 360$.
 (b) Find the coordinates of the minimum turning point P.

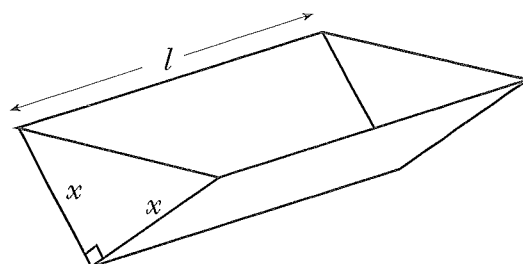


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8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



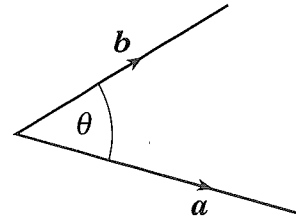
The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



- (a) Show that the surface area to be lined, A cm², is given by $A(x) = x^2 + \frac{432000}{x}$. 3
 (b) Find the value of x which minimises this surface area. 5

Marks

9. The diagram shows vectors \mathbf{a} and \mathbf{b} .
 If $|\mathbf{a}| = 5$, $|\mathbf{b}| = 4$ and $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 36$, find the
 size of the acute angle θ between \mathbf{a} and \mathbf{b} .



4

10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal
 places.

5

11. (a) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.
 (ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$.
 (b) Prove that the graphs intersect at a point where the x -coordinate is
 $\log_a\left(\frac{1}{a-1}\right)$.

2

3

[END OF QUESTION PAPER]

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