

X100/301

NATIONAL
QUALIFICATIONS
2005

FRIDAY, 20 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

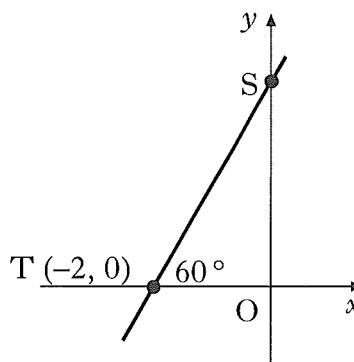
Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

1. Find the equation of the line ST, where T is the point $(-2, 0)$ and angle STO is 60° .



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2. Two congruent circles, with centres A and B, touch at P.

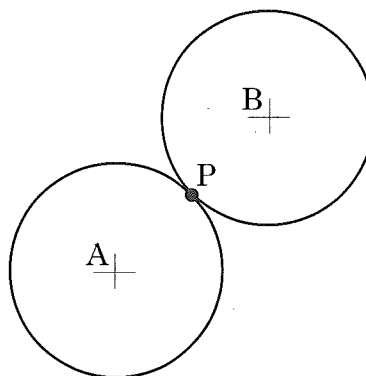
Relative to suitable axes, their equations are

$$x^2 + y^2 + 6x + 4y - 12 = 0 \text{ and}$$

$$x^2 + y^2 - 6x - 12y + 20 = 0.$$

- (a) Find the coordinates of P.

- (b) Find the length of AB.



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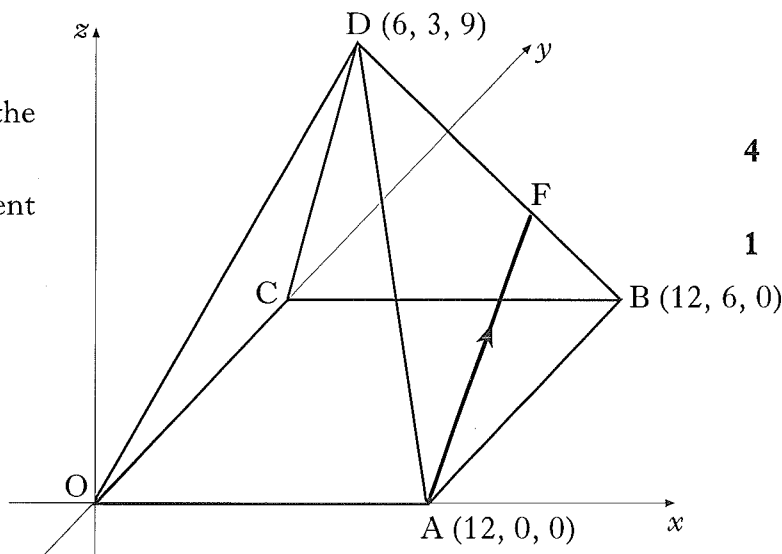
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3. D,OABC is a pyramid. A is the point $(12, 0, 0)$, B is $(12, 6, 0)$ and D is $(6, 3, 9)$.

F divides DB in the ratio 2:1.

- (a) Find the coordinates of the point F.

- (b) Express \vec{AF} in component form.



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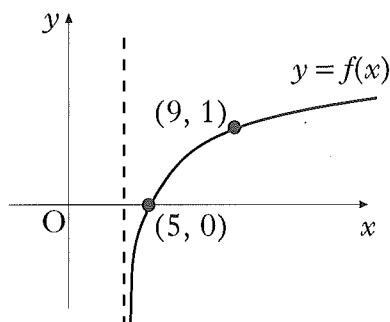
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4. Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.
- (a) Find $h(x)$ where $h(x) = g(f(x))$. 2
- (b) (i) Write down the coordinates of the minimum turning point of $y = h(x)$.
 (ii) Hence state the range of the function h . 2

5. Differentiate $(1 + 2 \sin x)^4$ with respect to x . 2

6. (a) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. 2
- (b) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.
- (i) Express u_1 and u_2 in terms of m .
 (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. 5

7. The function f is of the form $f(x) = \log_b(x - a)$.
 The graph of $y = f(x)$ is shown in the diagram.



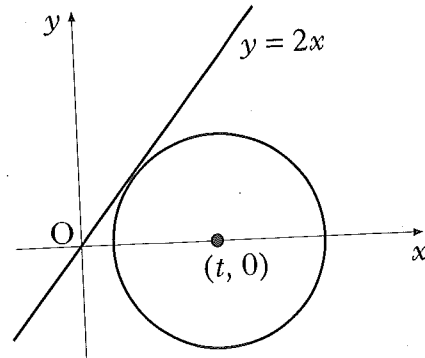
- (a) Write down the values of a and b . 2
- (b) State the domain of f . 1

8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.
- (a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. 5
- (b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x - and y -axes. 2
- (c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. 5

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$. 4

10. (a) Express $\sin x - \sqrt{3} \cos x$ in the form $k \sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$. 4
 (b) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3} \cos x$ in the interval $0 \leq x \leq 2\pi$. 5

11. (a) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.
 Write down the equation of the circle.
 (b) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle.



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X100/303

NATIONAL
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FRIDAY, 20 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

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Marks

1. Find $\int \frac{4x^3 - 1}{x^2} dx, x \neq 0.$

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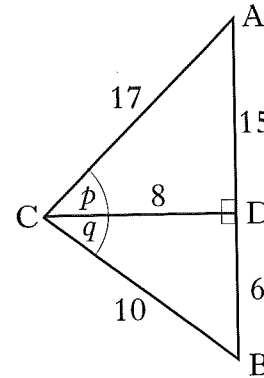
2. Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.

(a) Show that the exact value of $\sin(p + q)$ is $\frac{84}{85}.$

(b) Calculate the exact values of:

(i) $\cos(p + q);$

(ii) $\tan(p + q).$

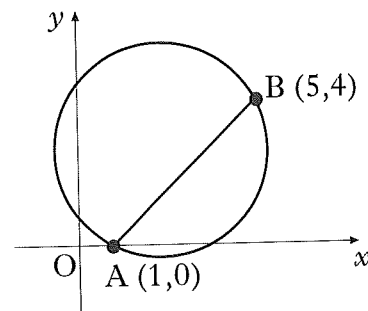


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3. (a) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

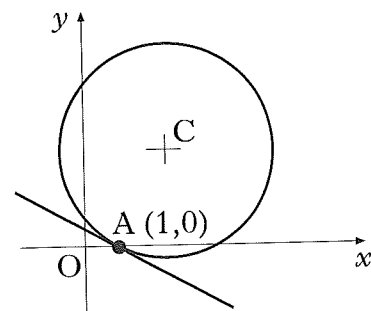
Show that the equation of the perpendicular bisector of chord AB is $x + y = 5.$



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(b) The point C is the centre of this circle. The tangent at the point A on the circle has equation $x + 3y = 1.$

Find the equation of the radius CA.



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(c) (i) Determine the coordinates of the point C.

(ii) Find the equation of the circle.

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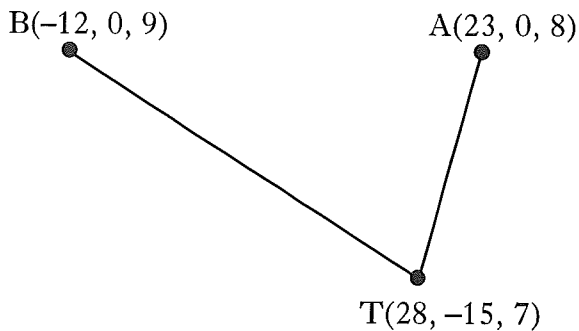
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Marks

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- (a) Express the vectors \vec{TA} and \vec{TB} in component form.

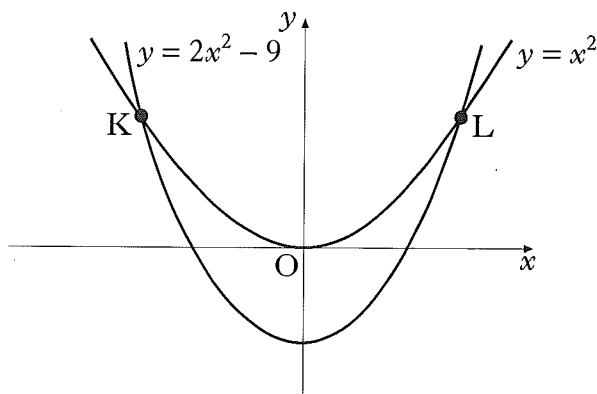
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- (b) Calculate the angle between these two beams.

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5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

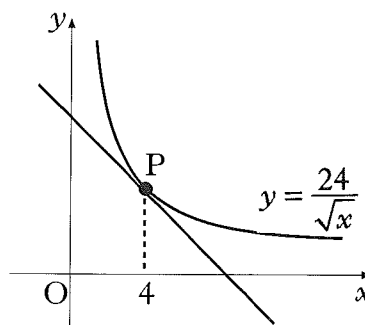
Calculate the area enclosed between the curves.



8

6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$.



6

7. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$.

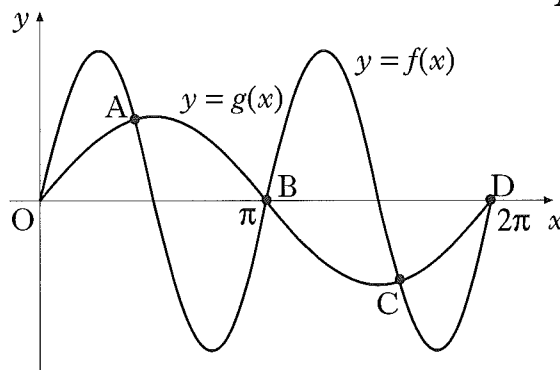
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Marks

8. Two functions, f and g , are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where $k > 1$.

The diagram shows the graphs of $y = f(x)$ and $y = g(x)$ intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \frac{1}{2k}$.



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9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$.

(a) What was its value when launched?

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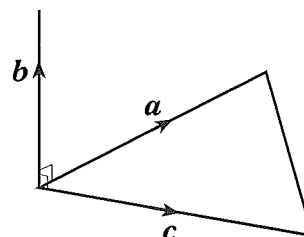
(b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?

4

10. Vectors a and c are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector b is 2 units long and b is perpendicular to both a and c .

Evaluate the scalar product $a \cdot (a + b + c)$.



4

11. (a) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$.
- (b) Hence find the range of values of p for which all the roots of the cubic equation are real.

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[END OF QUESTION PAPER]

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