

# **X100/301**

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NATIONAL  
QUALIFICATIONS  
2007

TUESDAY, 15 MAY  
9.00 AM – 10.10 AM

**MATHEMATICS  
HIGHER**

Units 1, 2 and 3

Paper 1

(Non-calculator)

## **Read Carefully**

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

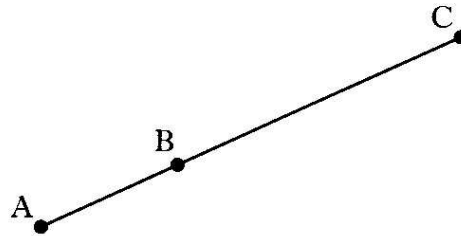
1. Find the equation of the line through the point  $(-1, 4)$  which is parallel to the line with equation  $3x - y + 2 = 0$ .

3

2. Relative to a suitable coordinate system A and B are the points  $(-2, 1, -1)$  and  $(1, 3, 2)$  respectively.

A, B and C are collinear points and C is positioned such that  $BC = 2AB$ .

Find the coordinates of C.



4

3. Functions  $f$  and  $g$ , defined on suitable domains, are given by  $f(x) = x^2 + 1$  and  $g(x) = 1 - 2x$ .

Find:

(a)  $g(f(x))$ ;

2

(b)  $g(g(x))$ .

2

4. Find the range of values of  $k$  such that the equation  $kx^2 - x - 1 = 0$  has no real roots.

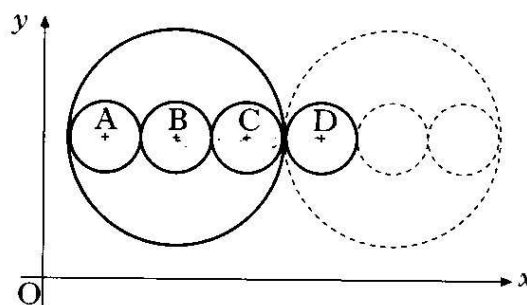
4

5. The large circle has equation  $x^2 + y^2 - 14x - 16y + 77 = 0$ .

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the  $x$ -axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



5

[Turn over

6. Solve the equation  $\sin 2x^\circ = 6\cos x^\circ$  for  $0 \leq x \leq 360$ . 4

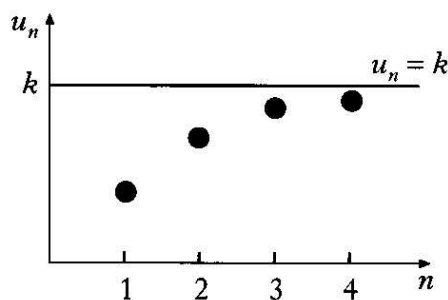
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, u_0 = 0.$$

(a) Calculate the values of  $u_1, u_2$  and  $u_3$ . 3

Four terms of this sequence,  $u_1, u_2, u_3$  and  $u_4$  are plotted as shown in the graph.

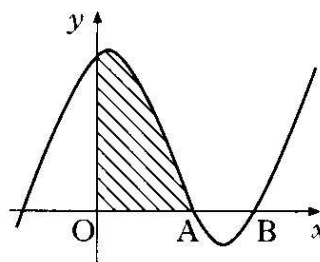
As  $n \rightarrow \infty$ , the points on the graph approach the line  $u_n = k$ , where  $k$  is the limit of this sequence.



(b) (i) Give a reason why this sequence has a limit. 3  
 (ii) Find the exact value of  $k$ .

8. The diagram shows a sketch of the graph of  $y = x^3 - 4x^2 + x + 6$ .

(a) Show that the graph cuts the  $x$ -axis at  $(3, 0)$ . 1  
 (b) Hence or otherwise find the coordinates of A. 3  
 (c) Find the shaded area. 5



9. A function  $f$  is defined by the formula  $f(x) = 3x - x^3$ .  
 (a) Find the exact values where the graph of  $y = f(x)$  meets the  $x$ - and  $y$ -axes. 2  
 (b) Find the coordinates of the stationary points of the function and determine their nature. 7  
 (c) Sketch the graph of  $y = f(x)$ . 1

10. Given that  $y = \sqrt{3x^2 + 2}$ , find  $\frac{dy}{dx}$ . 3
11. (a) Express  $f(x) = \sqrt{3} \cos x + \sin x$  in the form  $k \cos(x - a)$ , where  $k > 0$  and  $0 < a < \frac{\pi}{2}$ . 4
- (b) Hence or otherwise sketch the graph of  $y = f(x)$  in the interval  $0 \leq x \leq 2\pi$ . 4

[END OF QUESTION PAPER]

# **X100/303**

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NATIONAL  
QUALIFICATIONS  
2007

TUESDAY, 15 MAY  
10.30 AM – 12.00 NOON

**MATHEMATICS**  
**HIGHER**  
Units 1, 2 and 3  
Paper 2

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## FORMULAE LIST

### Circle:

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**Trigonometric formulae:**

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

**Table of standard derivatives:**

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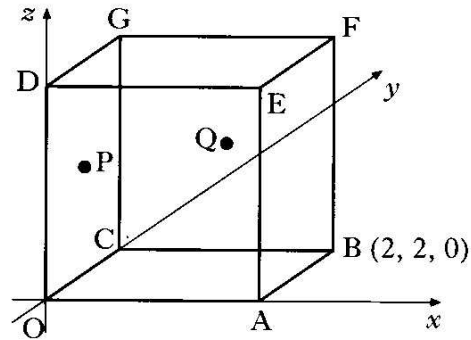
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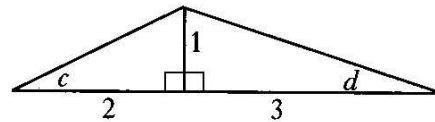
Marks

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.  
 B has coordinates (2, 2, 0).  
 P is the centre of face OCGD and Q is the centre of face CBFG.



- (a) Write down the coordinates of G. 1  
 (b) Find  $\mathbf{p}$  and  $\mathbf{q}$ , the position vectors of points P and Q. 2  
 (c) Find the size of angle POQ. 5

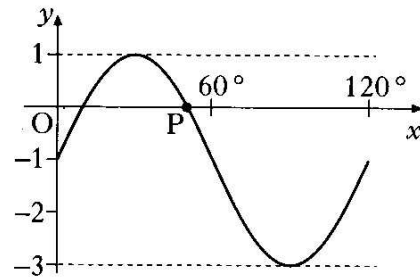
2. The diagram shows two right-angled triangles with angles  $c$  and  $d$  marked as shown.



- (a) Find the exact value of  $\sin(c + d)$ . 4  
 (b) (i) Find the exact value of  $\sin 2c$ . 4  
 (ii) Show that  $\cos 2d$  has the same exact value. 4

3. Show that the line with equation  $y = 6 - 2x$  is a tangent to the circle with equation  $x^2 + y^2 + 6x - 4y - 7 = 0$  and find the coordinates of the point of contact of the tangent and the circle. 6

4. The diagram shows part of the graph of a function whose equation is of the form  $y = a \sin(bx^\circ) + c$ .



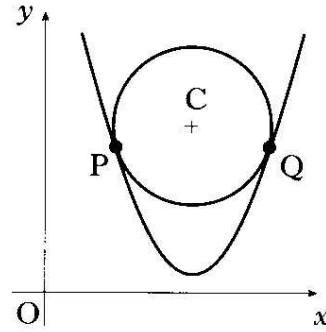
- (a) Write down the values of  $a$ ,  $b$  and  $c$ . 3  
 (b) Determine the exact value of the  $x$ -coordinate of P, the point where the graph intersects the  $x$ -axis as shown in the diagram. 3

[Turn over



5. A circle centre C is situated so that it touches the parabola with equation  $y = \frac{1}{2}x^2 - 8x + 34$  at P and Q.

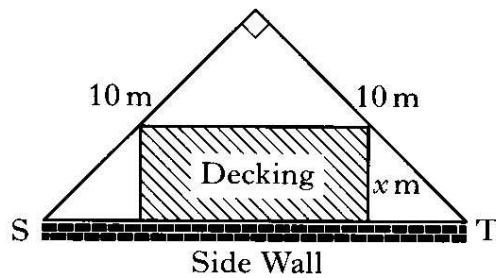
- (a) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q.
- (b) Find the coordinates of P.
- (c) Find the coordinates of C, the centre of the circle.



5  
2  
2

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



- (a) (i) Find the exact value of ST.
- (ii) Given that the breadth of the decking is  $x$  metres, show that the area of the decking,  $A$  square metres, is given by

$$A = (10\sqrt{2})x - 2x^2. \quad 3$$

- (b) Find the dimensions of the decking which maximises its area. 5

7. Find the value of  $\int_0^2 \sin(4x+1) dx$ . 4

8. The curve with equation  $y = \log_3(x-1) - 2 \cdot 2$ , where  $x > 1$ , cuts the  $x$ -axis at the point  $(a, 0)$ .

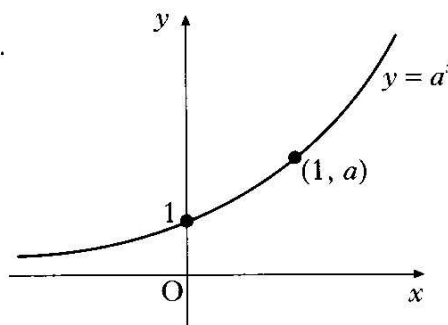
Find the value of  $a$ . 4

9. The diagram shows the graph of  $y = a^x$ ,  $a > 1$ .

On separate diagrams, sketch the graphs of:

(a)  $y = a^{-x}$ ;

(b)  $y = a^{1-x}$ .

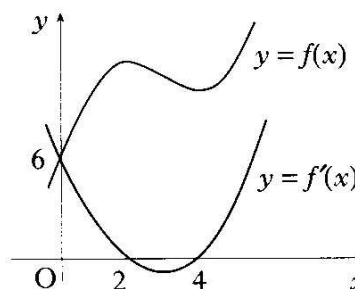


2  
2

10. The diagram shows the graphs of a cubic function  $y = f(x)$  and its derived function  $y = f'(x)$ .

Both graphs pass through the point  $(0, 6)$ .

The graph of  $y = f'(x)$  also passes through the points  $(2, 0)$  and  $(4, 0)$ .



(a) Given that  $f'(x)$  is of the form  $k(x - a)(x - b)$ :

(i) write down the values of  $a$  and  $b$ ;

(ii) find the value of  $k$ .

3

(b) Find the equation of the graph of the cubic function  $y = f(x)$ .

4

11. Two variables  $x$  and  $y$  satisfy the equation  $y = 3 \times 4^x$ .

(a) Find the value of  $a$  if  $(a, 6)$  lies on the graph with equation  $y = 3 \times 4^x$ .

1

(b) If  $(-\frac{1}{2}, b)$  also lies on the graph, find  $b$ .

1

(c) A graph is drawn of  $\log_{10}y$  against  $x$ . Show that its equation will be of the form  $\log_{10}y = Px + Q$  and state the gradient of this line.

4

[END OF QUESTION PAPER]