X100/301

NATIONAL QUALIFICATIONS 2008

TUESDAY, 20 MAY 9.00 AM – 10.30 AM

MATHEMATICS HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)
Instructions for completion of **Section A** are given on page two.
For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation

$$u_{n+1} = 0{\cdot}3u_n + 6 \text{ with } u_{10} = 10.$$

What is the value of u_{12}?

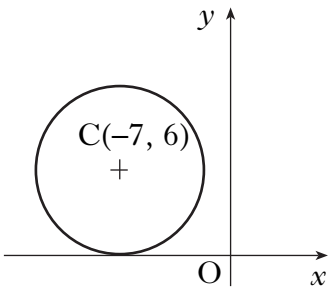
A 6·6

B 7·8

C 8·7

D 9·6

2. The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.



What is the equation of the circle?

A $(x + 7)^2 + (y - 6)^2 = 1$

B $(x + 7)^2 + (y - 6)^2 = 49$

C $(x - 7)^2 + (y + 6)^2 = 36$

D $(x + 7)^2 + (y - 6)^2 = 36$

3. The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular.

What is the value of k?

A 0

B 3

C 4

D 5

4. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$.

What is the limit of this sequence as $n \to \infty$?

A -800

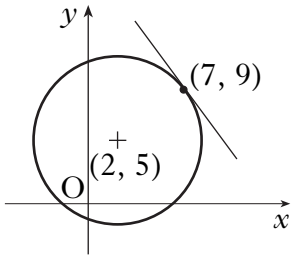
B -400

C 200

D 400

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

[Turn over

6. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$?

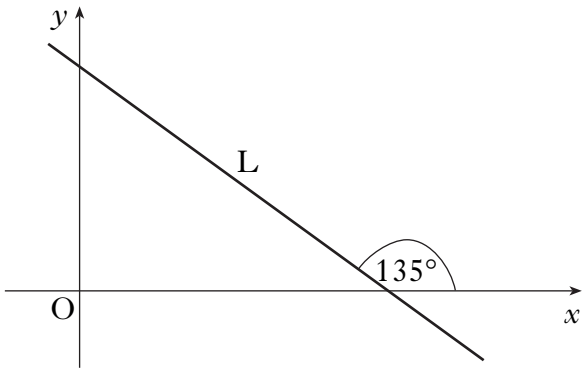
A $\frac{\pi}{6}$

B $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D $\frac{5\pi}{6}$

7. The diagram shows a line L; the angle between L and the positive direction of the x-axis is 135°, as shown.



What is the gradient of line L?

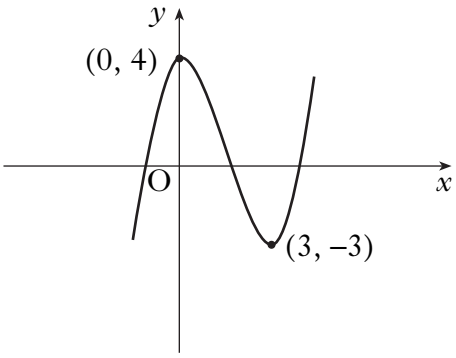
A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C -1

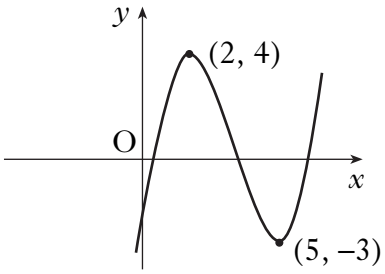
D $\frac{1}{2}$

8. The diagram shows part of the graph of a function with equation $y = f(x)$.

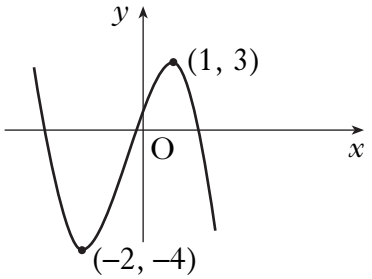


Which of the following diagrams shows the graph with equation $y = -f(x - 2)$?

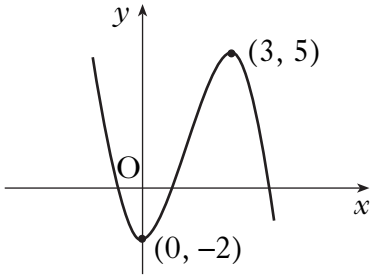
A



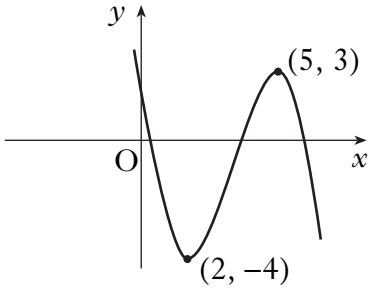
B



C



D



9. Given that $0 \leq a \leq \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.

A $\sin x + \frac{3}{5}$

B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$

C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$

D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$

10. Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

11. E(–2, –1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.

P lies between E and F.

What is the ratio in which P divides EF?

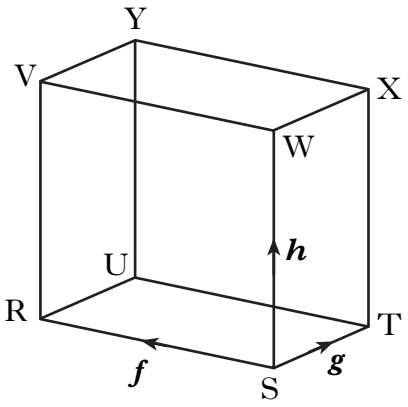
A 1:1

B 1:2

C 1:4

D 1:6

12. In the diagram RSTU, VWXY represents a cuboid.
$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.
Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.



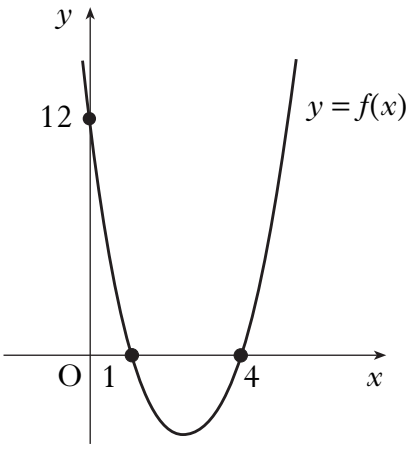
A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$

B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$

D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

13. The diagram shows part of the graph of a quadratic function $y = f(x)$.
The graph has an equation of the form $y = k(x - a)(x - b)$.



What is the equation of the graph?

A $y = 3(x - 1)(x - 4)$

B $y = 3(x + 1)(x + 4)$

C $y = 12(x - 1)(x - 4)$

D $y = 12(x + 1)(x + 4)$

14. Find $\int 4\sin(2x + 3)\, dx$.

A $-4\cos(2x + 3) + c$

B $-2\cos(2x + 3) + c$

C $4\cos(2x + 3) + c$

D $8\cos(2x + 3) + c$

15. What is the derivative of $(x^3 + 4)^2$?

A $(3x^2 + 4)^2$

B $\frac{1}{3}(x^3 + 4)^3$

C $6x^2(x^3 + 4)$

D $2(3x^2 + 4)^{-1}$

16. $2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of q?

A 5

B 7

C 9

D 11

17. A function f is given by $f(x) = \sqrt{9 - x^2}$.

What is a suitable domain of f?

A $x \geq 3$

B $x \leq 3$

C $-3 \leq x \leq 3$

D $-9 \leq x \leq 9$

18. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$.

Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.

A 0

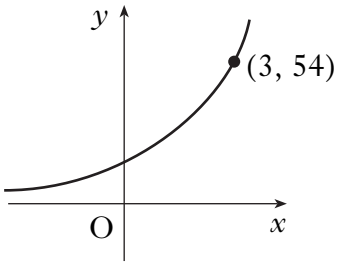
B 14

C 26

D 28

19. The diagram shows part of the graph whose equation is of the form $y = 2m^x$.

What is the value of m?



A 2

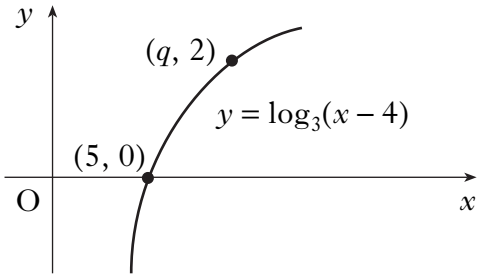
B 3

C 8

D 18

20. The diagram shows part of the graph of $y = \log_3(x - 4)$.

The point $(q, 2)$ lies on the graph.



What is the value of q?

A 6

B 7

C 8

D 13

[*END OF SECTION A*]

SECTION B

ALL questions should be attempted.

Marks

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

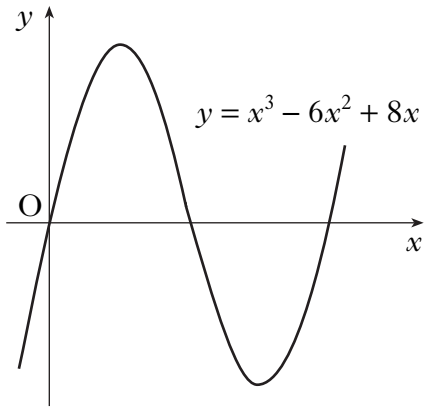
(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is –1. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

23. Functions f, g and h are defined on suitable domains by

$f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

(*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

(*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

X100/302

NATIONAL QUALIFICATIONS 2008

TUESDAY, 20 MAY 10.50 AM – 12.00 NOON

MATHEMATICS HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

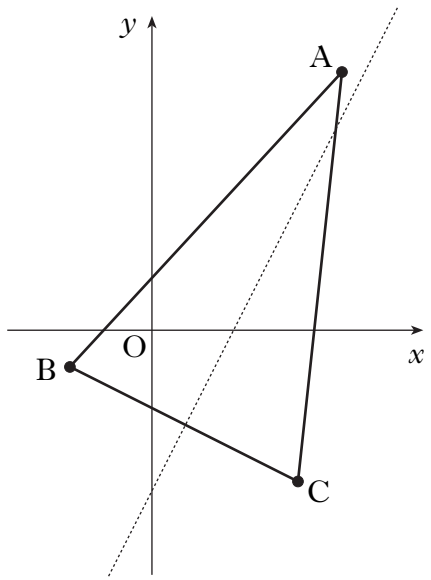
2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

ALL questions should be attempted.

Marks

1. The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.



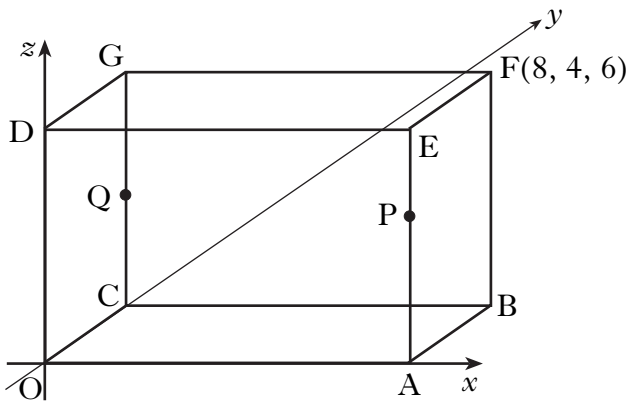
The broken line represents the perpendicular bisector of BC.

(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. **4**

(*b*) Find the equation of the median from C. **3**

(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. **3**

2. The diagram shows a cuboid OABC, DEFG.



F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.

(*a*) State the coordinates of P and Q. **2**

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. **2**

(*c*) Find the size of angle QPA. **5**

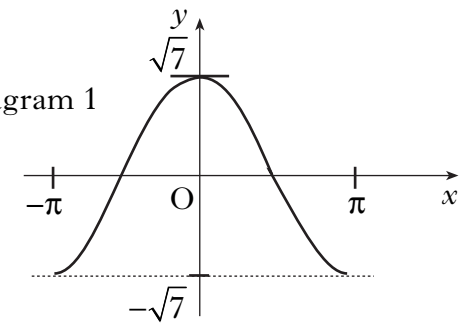
[Turn over

Marks

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

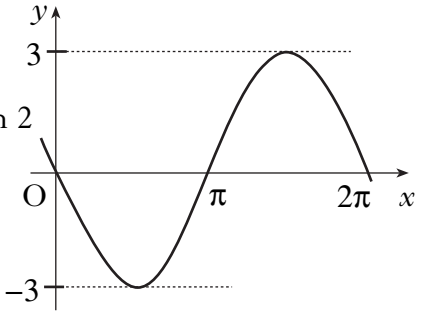
Diagram 1



(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q. **2**

Diagram 2



(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

4. (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. **2**

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. **4**

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

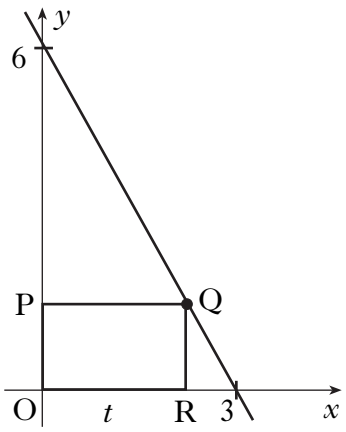
Find the coordinates of the points of intersection of the two circles. **5**

5. Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \leq x < 360$. **5**

Marks

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.



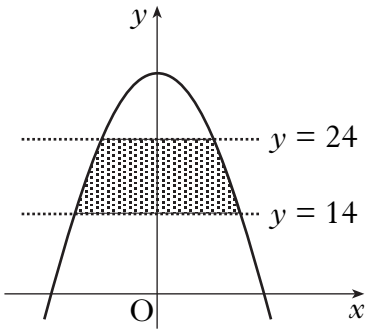
(*a*) Show that QR = $6 - 2t$. **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**

7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.



Calculate the shaded area. **8**

[*END OF QUESTION PAPER*]