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National
Qualifications
2022

Mark

X857/77/01 Physics

FRIDAY, 13 MAY

9:00 AM – 12:00 NOON

X85777 01

Fill in these boxes and read what is printed below.

Full name of centre

Town

Forename(s)

Surname

Number of seat

Date of birth

Day Month Year

Scottish candidate number

Total marks — 155

Attempt ALL questions.

Reference may be made to the Physics relationships sheet X857/77/11 and the data sheet on *page 02*.

Write your answers clearly in the spaces provided in this booklet. Additional space for answers and rough work is provided at the end of this booklet. If you use this space you must clearly identify the question number you are attempting. Any rough work must be written in this booklet. You should score through your rough work when you have written your final copy.

Care should be taken to give an appropriate number of significant figures in the final answers to calculations.

Use **blue** or **black** ink.

Before leaving the examination room you must give this booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

SQA ©

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DATA SHEET

COMMON PHYSICAL QUANTITIES

Quantity	Symbol	Value	Quantity	Symbol	Value
Gravitational acceleration on Earth	g	$9.8\ \text{m s}^{-2}$	Mass of electron	m_e	9.11×10^{-31} kg
Radius of Earth	R_E	6.4×10^{6} m	Charge on electron	e	-1.60×10^{-19} C
Mass of Earth	M_E	6.0×10^{24} kg	Mass of neutron	m_n	1.675×10^{-27} kg
Mass of Moon	M_M	7.3×10^{22} kg	Mass of proton	m_p	1.673×10^{-27} kg
Radius of Moon	R_M	1.7×10^{6} m	Mass of alpha particle	m_α	6.645×10^{-27} kg
Mean Radius of Moon Orbit		3.84×10^{8} m	Charge on alpha particle		3.20×10^{-19} C
Solar radius		6.955×10^{8} m	Charge on copper nucleus		4.64×10^{-18} C
Mass of Sun		2.0×10^{30} kg	Planck's constant	h	6.63×10^{-34} J s
1 AU		1.5×10^{11} m	Permittivity of free space	ε_0	$8.85 \times 10^{-12}\ \text{F m}^{-1}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}$	Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \text{H m}^{-1}$
Universal constant of gravitation	G	$6.67 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$	Speed of light in vacuum	c	$3.00 \times 10^{8}\ \text{m s}^{-1}$
			Speed of sound in air	v	$3.4 \times 10^{2}\ \text{m s}^{-1}$

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	Refractive index	Substance	Refractive index
Diamond	2.42	Glycerol	1.47
Glass	1.51	Water	1.33
Ice	1.31	Air	1.00
Perspex	1.49	Magnesium Fluoride	1.38

SPECTRAL LINES

Element	Wavelength (nm)	Colour
Hydrogen	656	Red
	486	Blue-green
	434	Blue-violet
	410	Violet
	397	Ultraviolet
	389	Ultraviolet
Sodium	589	Yellow
Cadmium	644	Red
	509	Green
	480	Blue

Lasers

Element	Wavelength (nm)	Colour
Carbon dioxide	9550 10 590	Infrared
Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	Density (kg m^{-3})	Melting Point (K)	Boiling Point (K)	Specific Heat Capacity ($\text{J kg}^{-1}\ \text{K}^{-1}$)	Specific Latent Heat of Fusion (J kg^{-1})	Specific Latent Heat of Vaporisation (J kg^{-1})
Aluminium	2.70×10^{3}	933	2623	9.02×10^{2}	3.95×10^{5}	. . . .
Copper	8.96×10^{3}	1357	2853	3.86×10^{2}	2.05×10^{5}	. . . .
Glass	2.60×10^{3}	1400	. . . .	6.70×10^{2}	. . . .	. . . .
Ice	9.20×10^{2}	273	. . . .	2.10×10^{3}	3.34×10^{5}	. . . .
Glycerol	1.26×10^{3}	291	563	2.43×10^{3}	1.81×10^{5}	8.30×10^{5}
Methanol	7.91×10^{2}	175	338	2.52×10^{3}	9.9×10^{4}	1.12×10^{6}
Sea Water	1.02×10^{3}	264	377	3.93×10^{3}	. . . .	. . . .
Water	1.00×10^{3}	273	373	4.18×10^{3}	3.34×10^{5}	2.26×10^{6}
Air	1.29	. . . .	. . . .	. . . .	. . . .	. . . .
Hydrogen	9.0×10^{-2}	14	20	1.43×10^{4}	. . . .	4.50×10^{5}
Nitrogen	1.25	63	77	1.04×10^{3}	. . . .	2.00×10^{5}
Oxygen	1.43	55	90	9.18×10^{2}	. . . .	2.40×10^{4}

The gas densities refer to a temperature of 273 K and a pressure of 1.01×10^{5} Pa.

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MARKS | DO NOT WRITE IN THIS MARGIN

Total marks — 155

Attempt ALL questions

1. During a short test run, a dragster accelerates from rest along a straight track.

The test run starts at time $t = 0$ s.

During the test run, the velocity v of the dragster at time t is given by the relationship

$$v = 6.6t^2 + 2.2t$$

where v is measured in m s^{-1} and t is measured in s.

(a) Using calculus methods:

(i) determine the acceleration of the dragster at $t = 4.1$ s **3**

Space for working and answer

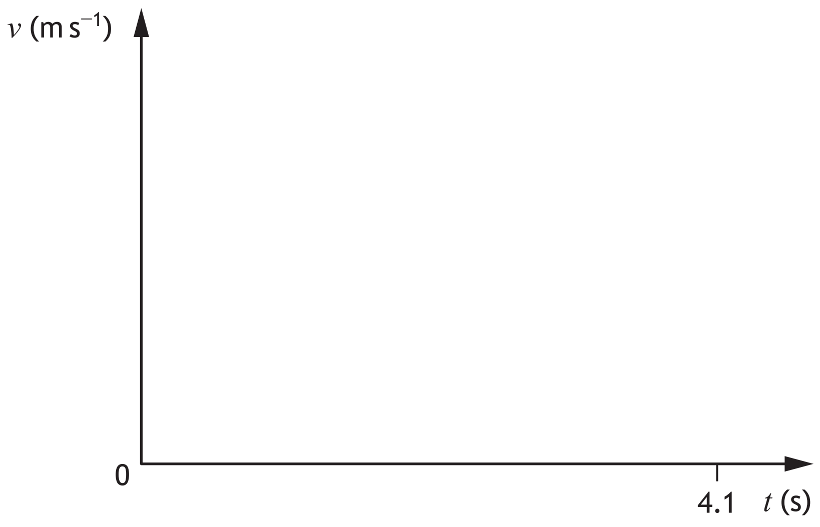
1. (a) (continued)

(ii) determine the distance travelled by the dragster between $t = 0$ s and $t = 4.1$ s. **3**

Space for working and answer

(b) On the axes below, sketch a graph to show the variation of velocity of the dragster with time, between $t = 0$ s and $t = 4.1$ s.

Numerical values are not required on the velocity axis. **1**



(An additional graph, if required, can be found on *page 54*.)

[Turn over

MARKS | DO NOT WRITE IN THIS MARGIN

2. A merry-go-round at a funfair rotates about an axis through its centre.

(a) The merry-go-round accelerates uniformly from rest. It takes 18 s to reach an angular velocity of 0.52 rad s^{-1}.

(i) Calculate the angular acceleration of the merry-go-round in this time. **3**

Space for working and answer

(ii) Calculate the angular displacement of the merry-go-round in this time. **3**

Space for working and answer

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2. (continued)

(b) Two students, X and Y, ride on the merry-go-round. The students are sitting on adjacent horses as shown in Figure 2A.

not to scale

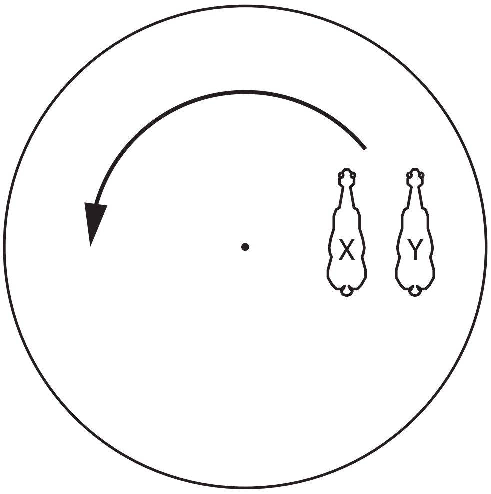

Figure 2A

(i) Explain why student Y has a greater tangential velocity than student X. **2**

(ii) State whether the centripetal acceleration of student Y is greater than, equal to, or less than the centripetal acceleration of student X.

You must justify your answer. **2**

3. A golf trolley consists of a frame with two identical wheels, as shown in Figure 3A. **MARKS** DO NOT WRITE IN THIS MARGIN

Figure 3A

Each wheel can be modelled as a hoop and five rods, as shown in Figure 3B.

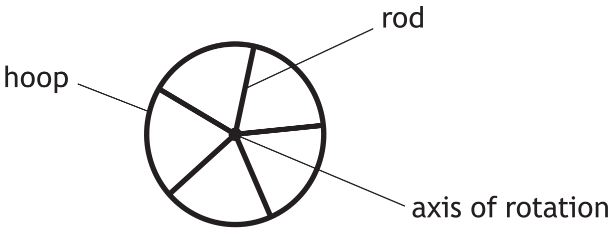

Figure 3B

The mass of the hoop is 0.38 kg. The radius of the hoop is 0.14 m.

The mass of each rod is 0.07 kg.

(a) Show that the moment of inertia of the wheel is 9.7×10^{-3} kg m^2. **3**

Space for working and answer

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3. (continued)

MARKS DO NOT WRITE IN THIS MARGIN

(b) A golfer cleans the wheels on the trolley by using a jet of air.

A wheel is raised off the ground. The jet of air exerts a tangential force of 1.2 N on the rim of the wheel as shown in Figure 3C. This causes the wheel to rotate.

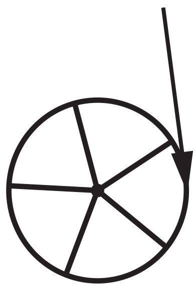

Figure 3C

(i) Calculate the torque acting on the wheel. **3**

Space for working and answer

(ii) A frictional torque also acts on the wheel.

When the 1.2 N force is applied, the wheel has an angular acceleration of 16 rad s^{-2}.

Determine the magnitude of the frictional torque. **4**

Space for working and answer

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[Turn over

3. (continued)

(c) The golfer now cleans the other wheel on the trolley. This wheel has a small stone stuck to the rim. The angular velocity of the wheel increases and the small stone 'flies off' the rim, as shown in Figure 3D.

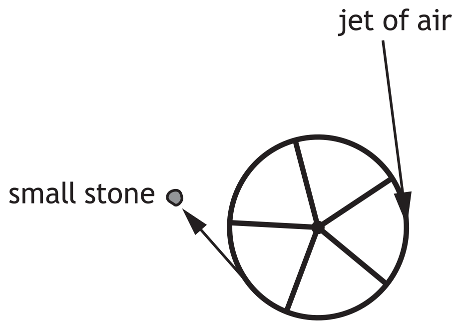


Figure 3D

Explain, in terms of forces, why the stone 'flies off' the rim. **2**

MARKS | DO NOT WRITE IN THIS MARGIN

4. A satellite of mass 2.30×10^3 kg is in a circular low Earth orbit.

The satellite orbits at an altitude of 312 km above the surface of the Earth, as shown in Figure 4A.

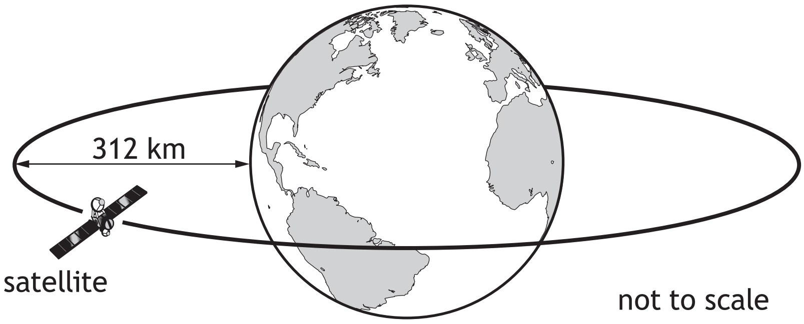


Figure 4A

(a) Show that the gravitational potential energy of the satellite in this orbit is -1.4×10^{11} J. **2**

Space for working and answer

(b) The satellite has an orbital period of 90.7 minutes.

Determine the speed of the satellite in this orbit. **3**

Space for working and answer

4. (continued)

(c) Determine the total energy of the satellite in this orbit. **3**

Space for working and answer

(d) Suggest why a satellite in a low-altitude orbit will lose energy at a greater rate than a similar satellite in a high-altitude orbit. **1**

[Turn over

4. (continued)

(e) The gravitational fields of the Earth and the Moon create five Lagrangian points.

A Lagrangian point is a position near two large bodies in orbit around each other, where a smaller object, such as a satellite, will remain in a fixed position relative to both orbiting bodies.

The distance r from the centre of the Moon to one of the Lagrangian points can be calculated using the relationship

$$r^3 = R^3\left(\frac{M_2}{3M_1}\right)$$

where R is the mean radius of the Moon's orbit

M_1 is the mass of the Earth

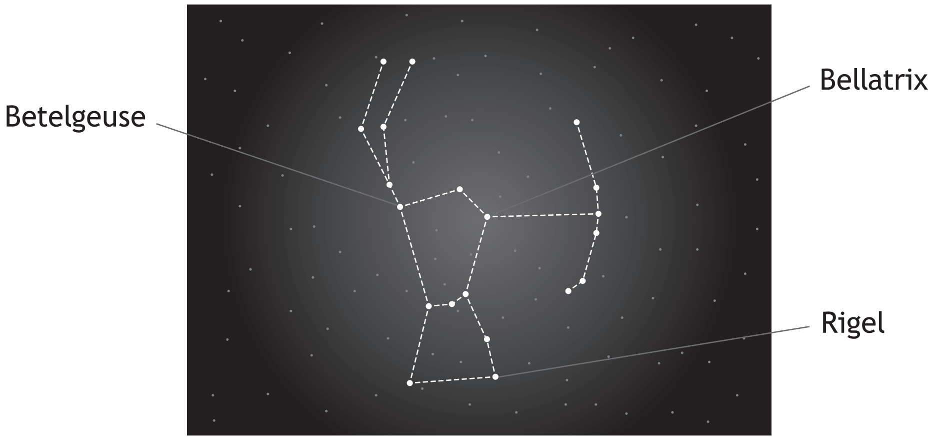
M_2 is the mass of the Moon.

Calculate the distance r from the centre of the Moon to this Lagrangian point. **2**

Space for working and answer

MARKS | DO NOT WRITE IN THIS MARGIN

5. Betelgeuse, Rigel, and Bellatrix are stars in the constellation Orion.



(a) Betelgeuse may ultimately become a black hole.

Betelgeuse has a mass of 2.19×10^{31} kg.

Calculate the Schwarzschild radius of Betelgeuse. **3**

Space for working and answer

(b) Rigel is no longer a main sequence star.

State the change that occurred in the fusion reactions within the core of Rigel at the point when it left the main sequence. **1**

5. (continued)

(c) Bellatrix is approximately 250 ly from Earth. It has a radius of 4.0×10^{9} m and an apparent brightness of 5.0×10^{-8} W m^{-2}.

Determine the surface temperature of Bellatrix. **5**

Space for working and answer

[Turn over

5. (continued)

(d) A group of students are discussing Rigel and Betelgeuse.

Student 1: 'Why does Rigel appear to have a blue-white colour, while Betelgeuse appears orange in colour?'

Student 2: 'Betelgeuse also looks brighter than Rigel, so it must be closer.'

Student 3: 'Betelgeuse and Rigel must be roughly the same distance from Earth, because they're in the same constellation.'

Student 4: 'I don't think Betelgeuse and Rigel are even in the same galaxy.'

Use your knowledge of physics to comment on the discussion. **3**

6. The Heisenberg uncertainty principle can be expressed as

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

(a) State an implication of this relationship for a quantum particle. **1**

(b) An alpha particle is emitted from a uranium-235 nucleus. According to classical physics, the alpha particle cannot overcome the strong nuclear force holding it in place in the nucleus.

Explain, in terms of the Heisenberg uncertainty principle, why alpha emission is possible from the uranium-235 nucleus. **2**

6. (continued)

(c) The mean lifetime of an alpha particle within the uranium-235 nucleus is 0.70 μs.

Determine the minimum uncertainty in the energy of this alpha particle. **3**

Space for working and answer

[Turn over

MARKS DO NOT WRITE IN THIS MARGIN

7. A student finds the diagram shown in Figure 7A in a textbook. The diagram represents some of the possible electron orbits in the Bohr model of an atom.

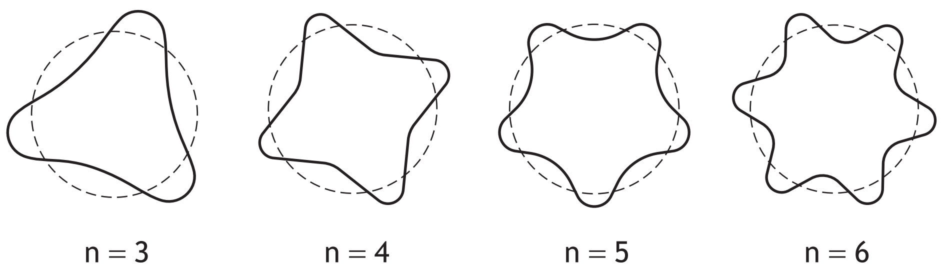


Figure 7A

Using your knowledge of physics, comment on the suitability of the diagram as a representation of electron orbits in an atom. **3**

MARKS | DO NOT WRITE IN THIS MARGIN

8. To produce an image of an atom, some microscopes use particles such as electrons or neutrons.

The de Broglie wavelengths of the particles should be approximately the same magnitude as, or smaller than, the diameter of the atom being imaged.

(a) In one electron microscope, the electrons used have a velocity of 1.75×10^7 m s^{-1}.

(i) Calculate the de Broglie wavelength of the electrons used. **3**

Space for working and answer

(ii) The diameter of an atom can be measured in ångströms (Å).

1 Å is equal to 0.1 nm.

The diameter of a gold atom is 2.6 Å.

(A) Explain whether electrons with velocity 1.75×10^7 m s^{-1} are suitable for imaging the gold atom. **1**

* X 8 5 7 7 7 0 1 2 4 *

8. (a) (ii) (continued)

(B) A neutron microscope uses neutrons with a velocity three orders of magnitude less than that of the electrons in the electron microscope.

Explain fully why the neutron microscope is suitable for imaging gold atoms. **2**

(b) Optical microscopes use visible light. Individual atoms are too small to be viewed using an optical microscope.

Estimate the diameter of the smallest object that could be imaged using an optical microscope. **1**

[Turn over

MARKS DO NOT WRITE IN THIS MARGIN

9. Charged particles originating from space approach the magnetic field of the Earth.

Most of the particles are high-energy protons.

A high-energy proton with a velocity of 2.75×10^{7} m s^{-1} enters the magnetic field of the Earth at a point where the magnetic induction is 23 μT. The proton enters the field at an angle of 60.0° and follows a helical path as shown in Figure 9A.

not to scale

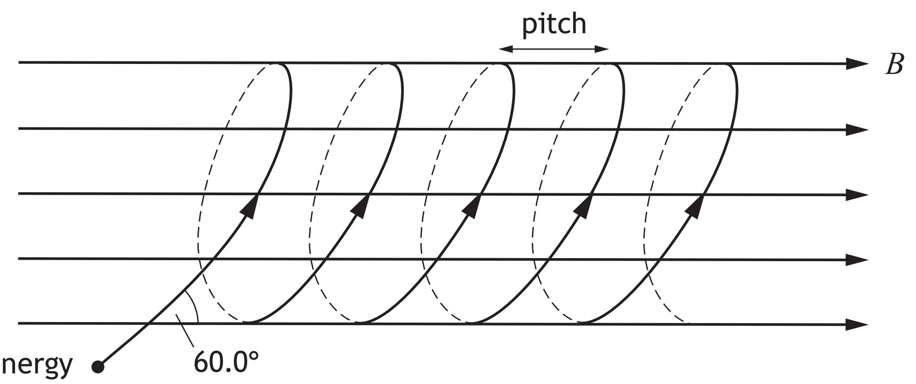


Figure 9A

(a) (i) Determine the component of the velocity of the proton parallel to the magnetic field. **1**

Space for working and answer

(ii) Determine the component of the velocity of the proton perpendicular to the magnetic field. **1**

Space for working and answer

9. **(continued)**

MARKS DO NOT WRITE IN THIS MARGIN

(b) (i) The component of the velocity of the proton perpendicular to the magnetic field causes it to experience a magnetic force.

Show that the magnetic force experienced by the proton in the magnetic field is 8.8×10^{-17} N. **2**

Space for working and answer

(ii) (A) This magnetic force causes the proton to undergo circular motion.

Calculate the radius of this circular motion. **3**

Space for working and answer

(B) Determine the period of this circular motion. **3**

Space for working and answer

9. (b) (continued) MARKS DO NOT WRITE IN THIS MARGIN

(iii) The distance that the proton moves parallel to the magnetic field lines during one period of the circular motion is known as the pitch.

Calculate the pitch of the helical path. **3**

Space for working and answer

(c) The magnetic induction increases closer to the poles, as shown in Figure 9B.

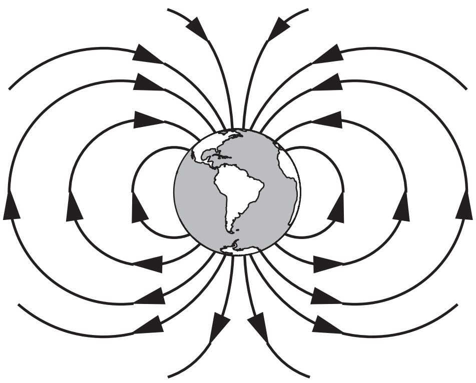


Figure 9B

The helical path of the proton follows a field line as it approaches the North Pole. The protons can be considered to be travelling at a constant speed.

Other than direction, state two changes to the helical path followed by the proton as it approaches the pole. **2**

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10. A student is studying simple harmonic motion (SHM) using a mass oscillating vertically on the end of a spring.

(a) State what is meant by *simple harmonic motion*. **1**

(b) The vertical displacement of an oscillating mass on a spring can be described by the expression

$$y = A\cos\left(\sqrt{\frac{k}{m}}\,t\right)$$

where the symbols have their usual meaning.

Show that this expression is a solution to the relationship

$$m\frac{d^2y}{dt^2} + ky = 0$$ **2**

10. (continued)

(c) A mass of 0.75 kg is suspended from a spring of negligible mass, as shown in Figure 10A.

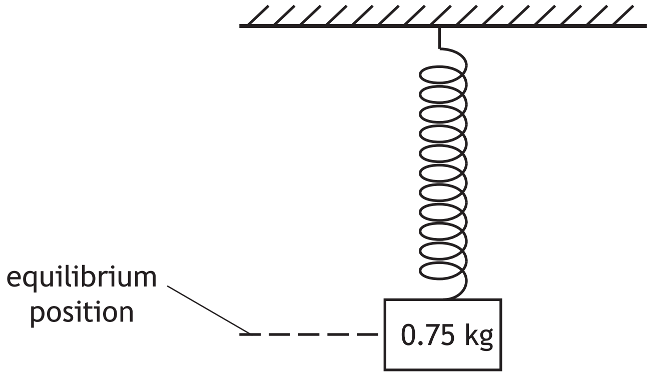


Figure 10A

The mass is now pulled down through a vertical distance of 0.038 m. It is then released, allowing it to oscillate about the equilibrium position.

The spring has a spring constant k of 24 N m^{-1}.

(i) By considering the expression

$$y = A\cos\left(\sqrt{\frac{k}{m}}\ t\right)$$

show that the angular frequency of the mass is 5.7 rad s^{-1}. **2**

Space for working and answer

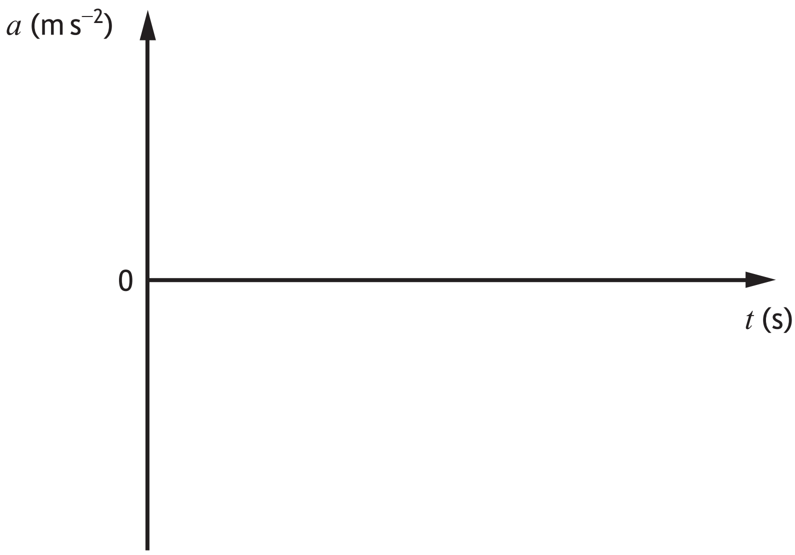
10. (c) (continued)

(ii) Determine the maximum acceleration of the mass. **3**

Space for working and answer

(iii) On the axes below, sketch a graph showing how the acceleration of the mass varies with time, for the first full oscillation.

Numerical values are required on the acceleration axis only. **2**



(An additional graph, if required, can be found on *page 54*.)

11. A travelling wave is represented by the equation

$$y = 12.6 \sin 2\pi(1.32t - 1.04x)$$

(a) The energy of the wave is 8.17 mJ.

The wave is reflected and its amplitude halves.

(i) Calculate the energy of this reflected wave. **3**

Space for working and answer

(ii) State the equation that represents this reflected wave. **2**

11. (continued)

(b) A graph of another travelling wave, at one instant in time, is shown in Figure 11A.

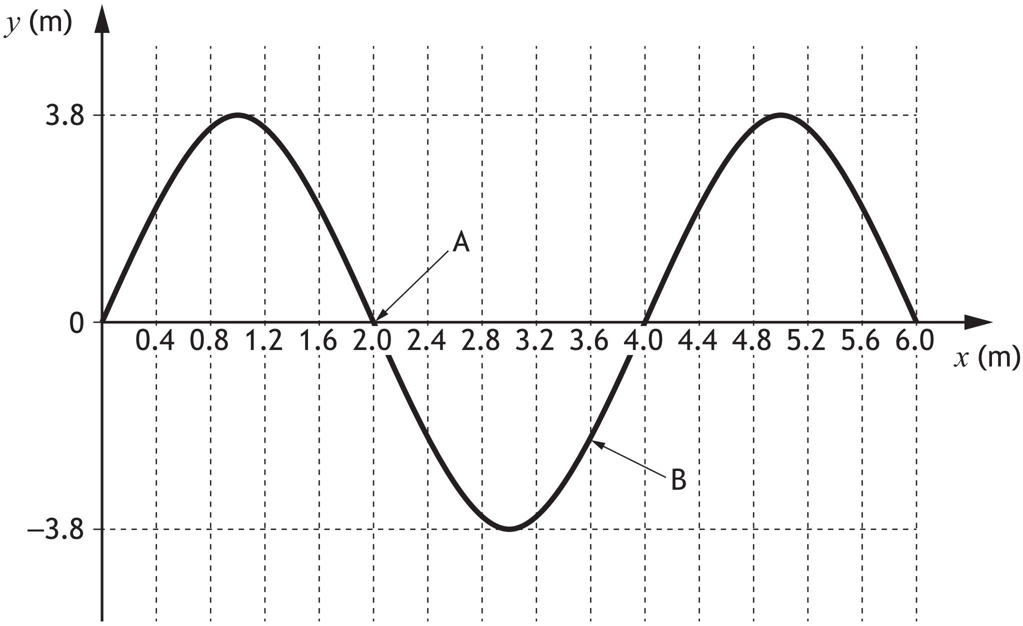


Figure 11A

Determine the phase difference between points A and B. **3**

Space for working and answer

12. A student carries out a Young's double slit experiment using a helium-neon laser. **MARKS** DO NOT WRITE IN THIS MARGIN

The student observes an interference pattern on the screen as shown in Figure 12A.

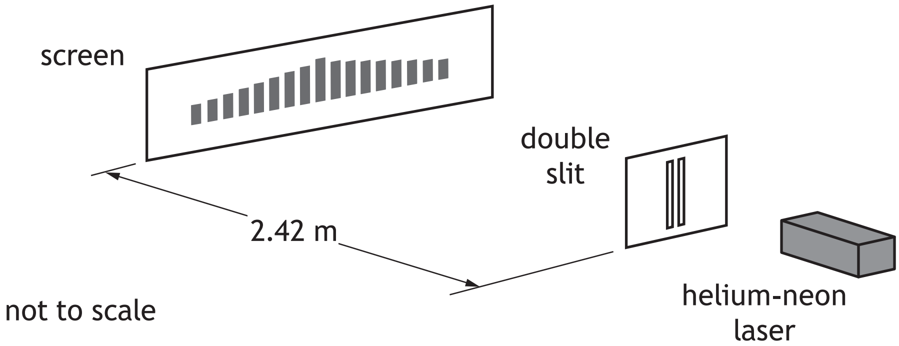


Figure 12A

(a) The student records their measurements.

Slit to screen distance (m)	Slit separation (mm)
2.42 ± 0.02	0.38 ± 0.01

(i) Using the student's measurements, calculate the fringe separation. **3**

Space for working and answer

(ii) Calculate the absolute uncertainty in this fringe separation. **3**

Space for working and answer

12. (continued)

(b) The student now measures across 16 fringe separations.

16 fringe separations = (62.4 ± 0.5) mm

Using this data, determine the fringe separation.

You must include an uncertainty in your answer. **1**

Space for working and answer

(c) State whether more confidence should be placed in the value for fringe separation obtained in (a) or in (b).

You must justify your answer. **2**

(d) The student now repeats the experiment using a laser that produces light of wavelength 532 nm.

State the effect this has on the fringe separation.

You must justify your answer. **2**

13. A student carries out an experiment to investigate the intensity of plane-polarised light transmitted through an analyser.

MARKS | DO NOT WRITE IN THIS MARGIN

(a) State what is meant by *plane-polarised light*. **1**

(b) The analyser can be rotated. The angle θ between the plane of polarisation and the transmission axis of the analyser is varied.

The light intensity is measured using a light meter.

This is shown in Figure 13A.

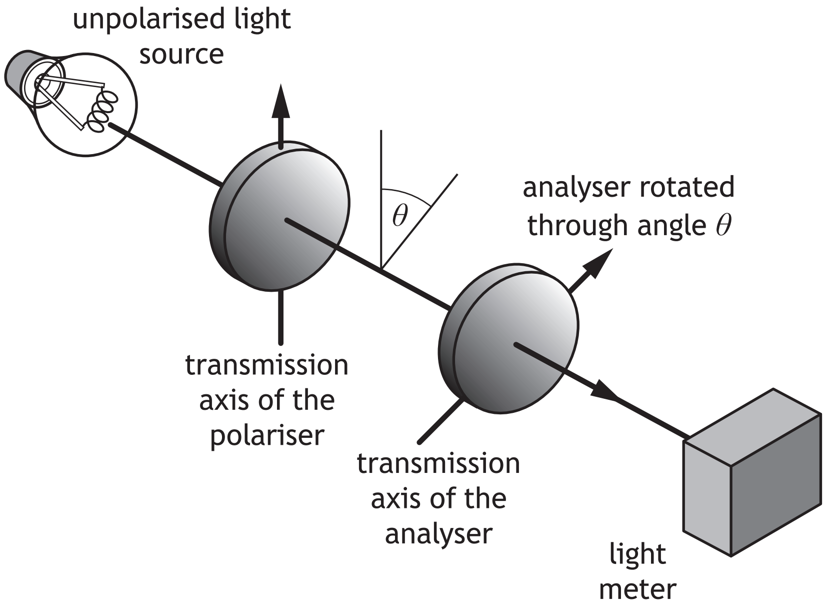


Figure 13A

The variation of measured light intensity I with θ is given by the relationship

$$I = I_0 \cos^2 \theta$$

where I_0 is the maximum light intensity.

Data from the student's experiment is shown in the table.

I (W m^{-2})	θ (°)	$\cos^2\theta$
4.0	30.0	0.75
3.2	40.0	
2.8	45.0	
1.6	60.0	
0.5	80.0	

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13. (b) (continued)

(i) Complete the table on *page 38* to show all derived values of $\cos^2\theta$. **1**

(ii) Using the square-ruled paper on *page 39*, draw a graph from which a value of I_0 can be determined.

(Additional square-ruled paper, if required, can be found on *pages 52* and *53*.) **3**

(iii) Use information from your graph to determine a value for I_0. **2**

(iv) Use information from your graph to determine the angle θ that gives a value for I of 3.5 W m^{-2}. **2**

(v) Use your graph to estimate the background light intensity. **1**

13. (continued)

(c) (i) Suggest one change to the **experimental procedure** that would improve the accuracy of measurements of light intensity. **1**

(ii) Suggest one change to the **experimental procedure** that would improve the precision of measurements of light intensity. **1**

[Turn over

14. In a cathode ray oscilloscope, electrons are accelerated from rest between the cathode and anode. The electrons then travel with a constant horizontal velocity between the parallel deflection plates.

This arrangement is shown in Figure 14A.

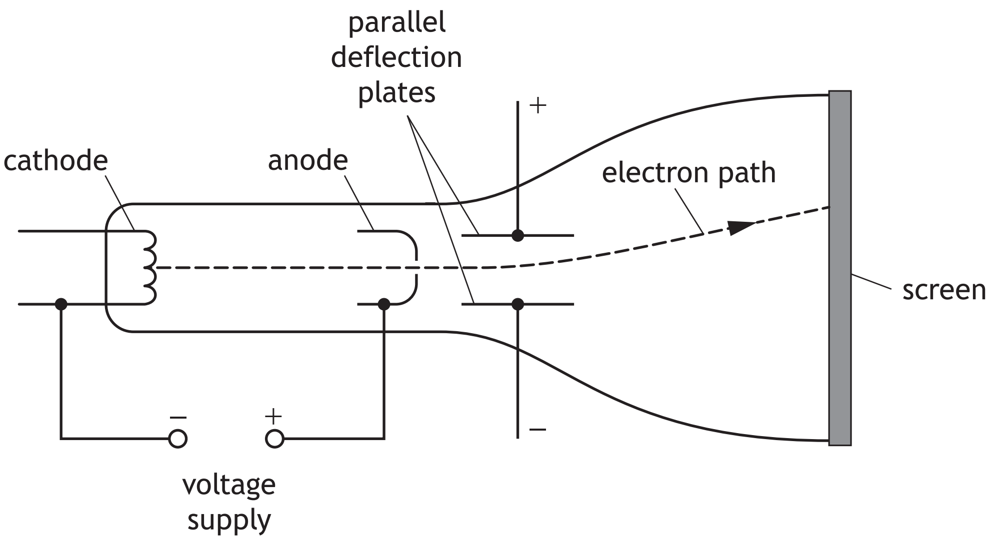


Figure 14A

(a) The electrons pass through the anode with a horizontal velocity of 2.9×10^{7} m s^{-1}.

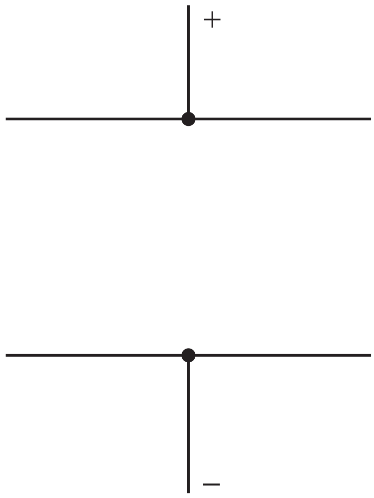
Determine the potential difference between the cathode and anode. **3**

Space for working and answer

MARKS | DO NOT WRITE IN THIS MARGIN

14. (continued)

(b) On the diagram below, sketch the electric field pattern between the parallel deflection plates. **1**



(An additional diagram, if required, can be found on *page 55*.)

(c) Explain why the electrons follow a curved path between the parallel deflection plates. **2**

[Turn over

14. (continued)

(d) The potential difference across the parallel deflection plates is 0.90 kV. Electrons passing between the plates are deflected by 4.0 mm in the vertical direction.

This is shown in Figure 14B.

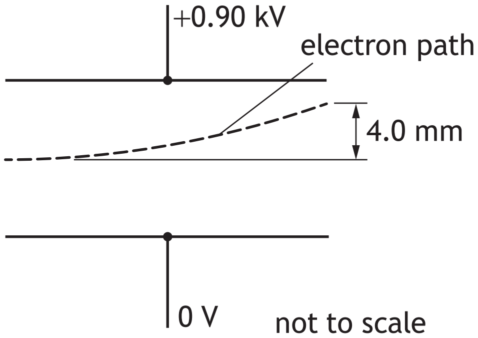


Figure 14B

(i) The vertical component of the velocity of the electrons is 1.2×10^{7} m s^{-1} as they exit the region between the plates.

Show that the vertical acceleration of the electrons between the parallel deflection plates is 1.8×10^{16} m s^{-2}. **2**

Space for working and answer

MARKS DO NOT WRITE IN THIS MARGIN

14. (d) (continued)

(ii) By considering the electric field between the plates, determine the vertical separation of the plates. **4**

Space for working and answer

[Turn over

MARKS | DO NOT WRITE IN THIS MARGIN

15. An undersea high voltage DC electrical power link consists of two cables buried under the seabed.

The magnetic permeability of the seabed can be taken to be the same as the permeability of free space.

There is a current of 1.80 kA in each cable.

The cables are buried 30.0 m apart, as shown in Figure 15A.

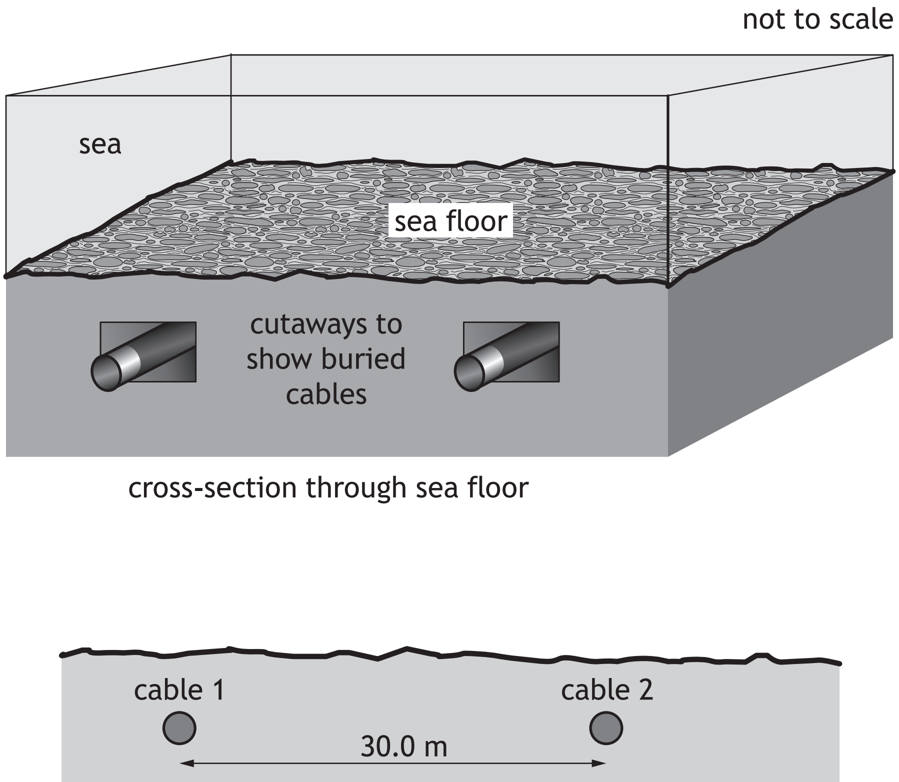


Figure 15A

(a) (i) Calculate the magnetic induction at cable 2 due to the current in cable 1. **3**

Space for working and answer

15. (a) (continued)

(ii) Determine the magnitude of the **force per unit length** acting on cable 2 due to the current in cable 1. 3

Space for working and answer

(b) A third cable carries a fibre-optic link. The optical fibre is made of silicon dioxide.

The speed v_m of an electromagnetic wave in an optical fibre is given by the relationship

$$v_m = \frac{1}{\sqrt{\varepsilon_r \, \varepsilon_0 \, \mu_r \, \mu_0}}$$

where ε_r is the relative permittivity of the optical fibre material

μ_r is the relative permeability of the optical fibre material

and the other symbols have their usual meaning.

The speed of light in the optical fibre is 1.52×10^8 m s^{-1}.

The relative permeability of silicon dioxide is 1.00.

Determine the relative permittivity of silicon dioxide. 2

Space for working and answer

16. An LC circuit in a radio receiver has an inductor and capacitor connected in parallel. The LC circuit is used to select different radio frequencies by varying the capacitance C of the capacitor.

The inductor has a fixed inductance L of 120 μH.

Part of the LC circuit is shown in Figure 16A.

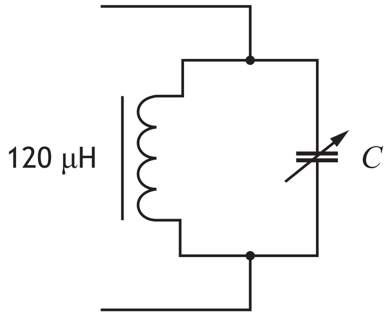


Figure 16A

(a) State what is meant by *inductive reactance*. **1**

(b) (i) The resonant frequency f_0 of the LC circuit is the frequency at which the inductive reactance equals the capacitive reactance.

Show that this frequency can be expressed as

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where the symbols have their usual meanings. **2**

Space for working and answer

MARKS DO NOT WRITE IN THIS MARGIN

16. (b) (continued)

(ii) The variation of the current with frequency in the LC circuit is shown in Figure 16B.

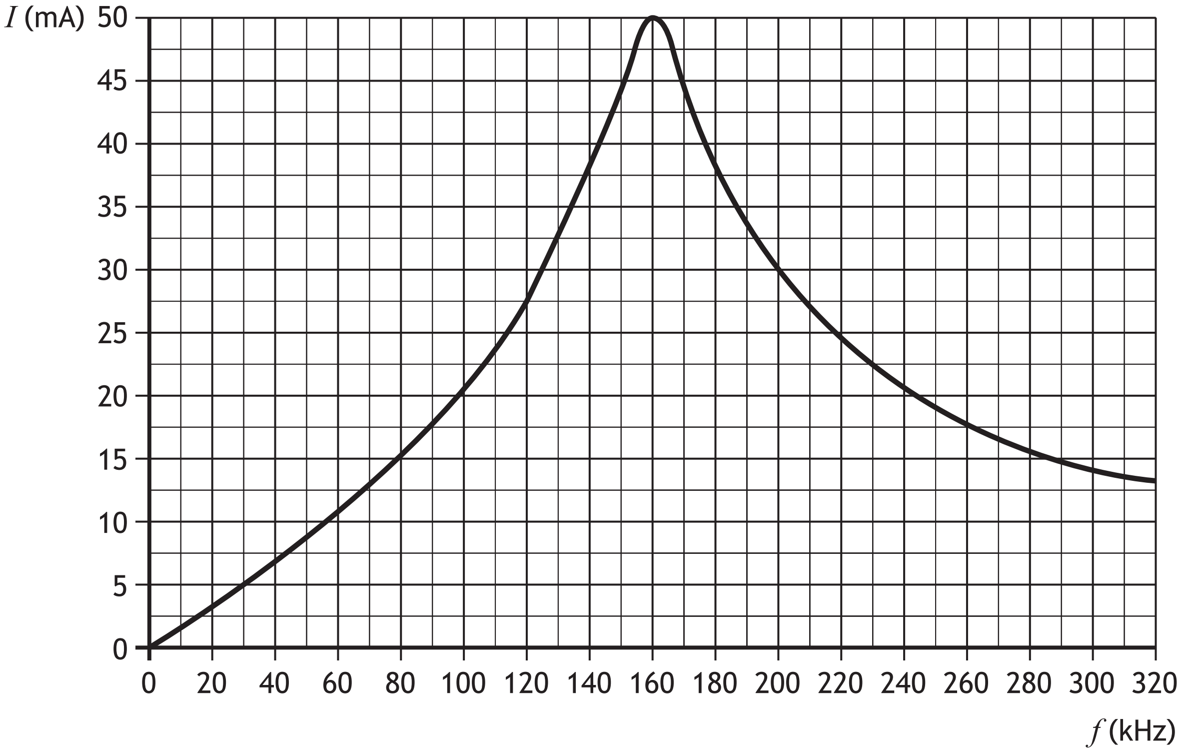


Figure 16B

At the resonant frequency, the current in the LC circuit is at a maximum.

Determine the capacitance of the capacitor at the resonant frequency. **3**

Space for working and answer

MARKS DO NOT WRITE IN THIS MARGIN

16. (continued)

(c) The radio receiver also contains an RC circuit. The RC circuit is shown in Figure 16C.

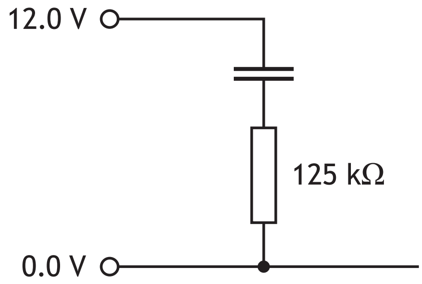


Figure 16C

The capacitor in the RC circuit is fully charged.

When the radio receiver is switched off, this capacitor discharges through a resistor of resistance 125 kΩ.

The time constant for the circuit is 250 s.

(i) Calculate the capacitance of this capacitor. **3**

Space for working and answer

16. (c) (continued)

(ii) A graph of the potential difference V across the capacitor against time t is shown in Figure 16D.

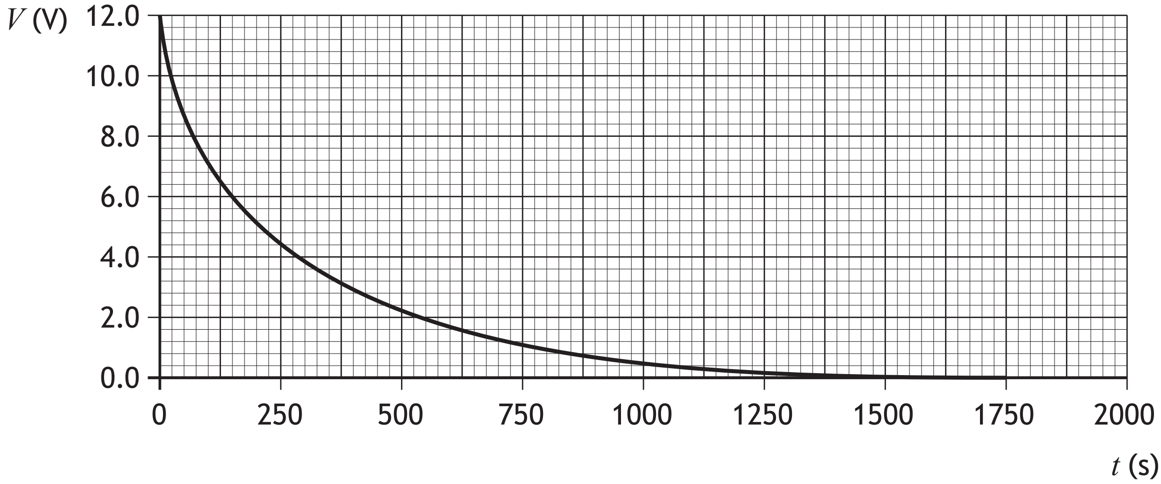


Figure 16D

Using information from the graph, show that the voltage across the capacitor reduces to 37% of its original value after one time constant. **2**

Space for working and answer

[END OF QUESTION PAPER]

AH National Qualifications 2022

X857/77/11

Physics
Relationships sheet

FRIDAY, 13 MAY

9:00 AM – 12:00 NOON

©

X857711

A/PB

Relationships required for Physics Advanced Higher

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 = \omega_o{}^2 + 2\alpha\theta$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$I = \sum mr^2$$

$$\tau = Fr$$

$$\tau = I\alpha$$

$$L = mvr = mr^2\omega$$

$$L = I\omega$$

$$E_{k(rotational)} = \frac{1}{2}I\omega^2$$

$$E_P = E_{k(translational)} + E_{k(rotational)}$$

$$F = \frac{GMm}{r^2}$$

$$F = \frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T}\right)^2$$

$$V = -\frac{GM}{r}$$

$$E_P = Vm = -\frac{GMm}{r}$$

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

$$r_{Schwarzschild} = \frac{2GM}{c^2}$$

$$b = \frac{L}{4\pi d^2}$$

$$\frac{P}{A} = \sigma T^4$$

$$L = 4\pi r^2\sigma T^4$$

$$E = hf$$

$$mvr = \frac{nh}{2\pi}$$

$$\lambda = \frac{h}{p}$$

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

$$F = qvB$$

$$F = \frac{mv^2}{r}$$

$$F = -ky$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y$$

$y = A\cos\omega t$ or $y = A\sin\omega t$

$$v = \pm\omega\sqrt{\left(A^2 - y^2\right)}$$

$$E_k = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$$

$$E_P = \frac{1}{2}m\omega^2 y^2$$

$$E = kA^2$$

$$y = A\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$$

$$\phi = \frac{2\pi x}{\lambda}$$

$$opd = n \times gpd$$

$opd = m\lambda$ or $\left(m + \frac{1}{2}\right)\lambda$ where $m = 0,1,2\ldots$

$$\Delta x = \frac{\lambda l}{2d}$$

$$d = \frac{\lambda}{4n}$$

$$\Delta x = \frac{\lambda D}{d}$$

$$n = \tan i_P$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2}$$

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$F = QE$$

$$V = Ed$$

$$W = QV$$

$$E_k = \frac{1}{2}mv^2$$

$$B = \frac{\mu_o I}{2\pi r}$$

$$F = IlB\sin\theta$$

$$F = qvB$$

$$\tau = RC$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{1}{2\pi fC}$$

$$\varepsilon = -L\frac{dI}{dt}$$

$$E = \frac{1}{2}LI^2$$

$$X_L = \frac{V}{I}$$

$$X_L = 2\pi fL$$

$$c = \frac{1}{\sqrt{\varepsilon_o\mu_o}}$$

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

$$\frac{\Delta W}{W} = \sqrt{\left(\frac{\Delta X}{X}\right)^2 + \left(\frac{\Delta Y}{Y}\right)^2 + \left(\frac{\Delta Z}{Z}\right)^2}$$

$$\left(\frac{\Delta W^n}{W^n}\right) = n\left(\frac{\Delta W}{W}\right)$$

$$d = \bar{v}t$$

$$s = \bar{v}t$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$W = mg$$

$$F = ma$$

$$E_W = Fd$$

$$E_P = mgh$$

$$E_K = \frac{1}{2}mv^2$$

$$P = \frac{E}{t}$$

$$p = mv$$

$$Ft = mv - mu$$

$$F = G\frac{Mm}{r^2}$$

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$f_o = f_s\left(\frac{v}{v \pm v_s}\right)$$

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{v}{c}$$

$$v = H_0 d$$

$$W = QV$$

$$E = mc^2$$

$$E = hf$$

$$E_K = hf - hf_0$$

$$E_2 - E_1 = hf$$

$$T = \frac{1}{f}$$

$$v = f\lambda$$

$$d \sin\theta = m\lambda$$

$$n = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\sin\theta_c = \frac{1}{n}$$

$$I = \frac{k}{d^2}$$

$$I = \frac{P}{A}$$

$$V_{peak} = \sqrt{2}V_{rms}$$

$$I_{peak} = \sqrt{2}I_{rms}$$

$$Q = It$$

$$V = IR$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$R_T = R_1 + R_2 + \ldots$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

$$E = V + Ir$$

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_S$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$C = \frac{Q}{V}$$

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

path difference $= m\lambda$ or $\left(m + \frac{1}{2}\right)\lambda$ where $m = 0, 1, 2\ldots$

random uncertainty $= \dfrac{\text{max. value} - \text{min. value}}{\text{number of values}}$

Additional relationships

Circle

circumference $= 2\pi r$

area $= \pi r^2$

Sphere

area $= 4\pi r^2$

volume $= \frac{4}{3}\pi r^3$

Trigonometry

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\sin^2\theta + \cos^2\theta = 1$

Moment of inertia

point mass
$I = mr^2$

rod about centre
$I = \frac{1}{12}ml^2$

rod about end
$I = \frac{1}{3}ml^2$

disc about centre
$I = \frac{1}{2}mr^2$

sphere about centre
$I = \frac{2}{5}mr^2$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Electron arrangements of elements

Key

Atomic number
Symbol
Electron arrangement
Name

Transition elements

Group 1 (1)	Group 2 (2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	Group 3 (13)	Group 4 (14)	Group 5 (15)	Group 6 (16)	Group 7 (17)	Group 0 (18)
1 **H** 1 Hydrogen																	2 **He** 2 Helium
3 **Li** 2,1 Lithium	4 **Be** 2,2 Beryllium											5 **B** 2,3 Boron	6 **C** 2,4 Carbon	7 **N** 2,5 Nitrogen	8 **O** 2,6 Oxygen	9 **F** 2,7 Fluorine	10 **Ne** 2,8 Neon
11 **Na** 2,8,1 Sodium	12 **Mg** 2,8,2 Magnesium											13 **Al** 2,8,3 Aluminium	14 **Si** 2,8,4 Silicon	15 **P** 2,8,5 Phosphorus	16 **S** 2,8,6 Sulfur	17 **Cl** 2,8,7 Chlorine	18 **Ar** 2,8,8 Argon
19 **K** 2,8,8,1 Potassium	20 **Ca** 2,8,8,2 Calcium	21 **Sc** 2,8,9,2 Scandium	22 **Ti** 2,8,10,2 Titanium	23 **V** 2,8,11,2 Vanadium	24 **Cr** 2,8,13,1 Chromium	25 **Mn** 2,8,13,2 Manganese	26 **Fe** 2,8,14,2 Iron	27 **Co** 2,8,15,2 Cobalt	28 **Ni** 2,8,16,2 Nickel	29 **Cu** 2,8,18,1 Copper	30 **Zn** 2,8,18,2 Zinc	31 **Ga** 2,8,18,3 Gallium	32 **Ge** 2,8,18,4 Germanium	33 **As** 2,8,18,5 Arsenic	34 **Se** 2,8,18,6 Selenium	35 **Br** 2,8,18,7 Bromine	36 **Kr** 2,8,18,8 Krypton
37 **Rb** 2,8,18,8,1 Rubidium	38 **Sr** 2,8,18,8,2 Strontium	39 **Y** 2,8,18,9,2 Yttrium	40 **Zr** 2,8,18, 10,2 Zirconium	41 **Nb** 2,8,18, 12,1 Niobium	42 **Mo** 2,8,18,13, 1 Molybdenum	43 **Tc** 2,8,18,13, 2 Technetium	44 **Ru** 2,8,18,15, 1 Ruthenium	45 **Rh** 2,8,18,16, 1 Rhodium	46 **Pd** 2,8,18, 18,0 Palladium	47 **Ag** 2,8,18, 18,1 Silver	48 **Cd** 2,8,18, 18,2 Cadmium	49 **In** 2,8,18, 18,3 Indium	50 **Sn** 2,8,18, 18,4 Tin	51 **Sb** 2,8,18, 18,5 Antimony	52 **Te** 2,8,18, 18,6 Tellurium	53 **I** 2,8,18, 18,7 Iodine	54 **Xe** 2,8,18, 18,8 Xenon
55 **Cs** 2,8,18,18, 8,1 Caesium	56 **Ba** 2,8,18,18, 8,2 Barium	57 **La** 2,8,18,18, 9,2 Lanthanum	72 **Hf** 2,8,18,32, 10,2 Hafnium	73 **Ta** 2,8,18, 32,11,2 Tantalum	74 **W** 2,8,18,32, 12,2 Tungsten	75 **Re** 2,8,18,32, 13,2 Rhenium	76 **Os** 2,8,18,32, 14,2 Osmium	77 **Ir** 2,8,18,32, 15,2 Iridium	78 **Pt** 2,8,18,32, 17,1 Platinum	79 **Au** 2,8,18, 32,18,1 Gold	80 **Hg** 2,8,18, 32,18,2 Mercury	81 **Tl** 2,8,18, 32,18,3 Thallium	82 **Pb** 2,8,18, 32,18,4 Lead	83 **Bi** 2,8,18, 32,18,5 Bismuth	84 **Po** 2,8,18, 32,18,6 Polonium	85 **At** 2,8,18, 32,18,7 Astatine	86 **Rn** 2,8,18, 32,18,8 Radon
87 **Fr** 2,8,18,32, 18,8,1 Francium	88 **Ra** 2,8,18,32, 18,8,2 Radium	89 **Ac** 2,8,18,32, 18,9,2 Actinium	104 **Rf** 2,8,18,32, 32,10,2 Rutherfordium	105 **Db** 2,8,18,32, 32,11,2 Dubnium	106 **Sg** 2,8,18,32, 32,12,2 Seaborgium	107 **Bh** 2,8,18,32, 32,13,2 Bohrium	108 **Hs** 2,8,18,32, 32,14,2 Hassium	109 **Mt** 2,8,18,32, 32,15,2 Meitnerium	110 **Ds** 2,8,18,32, 32,17,1 Darmstadtium	111 **Rg** 2,8,18,32, 32,18,1 Roentgenium	112 **Cn** 2,8,18,32, 32,18,2 Copernicium						

Lanthanides	57 **La** 2,8,18, 18,9,2 Lanthanum	58 **Ce** 2,8,18, 20,8,2 Cerium	59 **Pr** 2,8,18,21, 8,2 Praseodymium	60 **Nd** 2,8,18,22, 8,2 Neodymium	61 **Pm** 2,8,18,23, 8,2 Promethium	62 **Sm** 2,8,18,24, 8,2 Samarium	63 **Eu** 2,8,18,25, 8,2 Europium	64 **Gd** 2,8,18,25, 9,2 Gadolinium	65 **Tb** 2,8,18,27, 8,2 Terbium	66 **Dy** 2,8,18,28, 8,2 Dysprosium	67 **Ho** 2,8,18,29, 8,2 Holmium	68 **Er** 2,8,18,30, 8,2 Erbium	69 **Tm** 2,8,18,31, 8,2 Thulium	70 **Yb** 2,8,18,32, 8,2 Ytterbium	71 **Lu** 2,8,18,32, 9,2 Lutetium
Actinides	89 **Ac** 2,8,18,32, 18,9,2 Actinium	90 **Th** 2,8,18,32, 18,10,2 Thorium	91 **Pa** 2,8,18,32, 20,9,2 Protactinium	92 **U** 2,8,18,32, 21,9,2 Uranium	93 **Np** 2,8,18,32, 22,9,2 Neptunium	94 **Pu** 2,8,18,32, 24,8,2 Plutonium	95 **Am** 2,8,18,32, 25,8,2 Americium	96 **Cm** 2,8,18,32, 25,9,2 Curium	97 **Bk** 2,8,18,32, 27,8,2 Berkelium	98 **Cf** 2,8,18,32, 28,8,2 Californium	99 **Es** 2,8,18,32, 29,8,2 Einsteinium	100 **Fm** 2,8,18,32, 30,8,2 Fermium	101 **Md** 2,8,18,32, 31,8,2 Mendelevium	102 **No** 2,8,18,32, 32,8,2 Nobelium	103 **Lr** 2,8,18,32, 32,9,2 Lawrencium