

X272/13/01

NATIONAL TUESDAY, 5 MAY
QUALIFICATIONS 1.00 PM – 3.30 PM
2015

PHYSICS
ADVANCED HIGHER
(Revised)

Reference may be made to the Physics Data Booklet and the accompanying Relationships Sheet.

Answer **all** questions.

Any necessary data may be found in the Data Sheet on *Page two*.

Care should be taken to give an appropriate number of significant figures in the final answers to calculations.

Square-ruled paper (if used) should be placed inside the front cover of the answer book for return to the Scottish Qualifications Authority.



DATA SHEET
COMMON PHYSICAL QUANTITIES

<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>	<i>Quantity</i>	<i>Symbol</i>	<i>Value</i>
Gravitational acceleration on Earth	g	9.8 m s^{-2}	Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Radius of Earth	R_E	$6.4 \times 10^6 \text{ m}$	Charge on electron	e	$-1.60 \times 10^{-19} \text{ C}$
Mass of Earth	M_E	$6.0 \times 10^{24} \text{ kg}$	Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Mass of Moon	M_M	$7.3 \times 10^{22} \text{ kg}$	Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Radius of Moon	R_M	$1.7 \times 10^6 \text{ m}$	Mass of alpha particle	m_α	$6.645 \times 10^{-27} \text{ kg}$
Mean Radius of Moon Orbit		$3.84 \times 10^8 \text{ m}$	Charge on alpha particle		$3.20 \times 10^{-19} \text{ C}$
Solar radius		$6.955 \times 10^8 \text{ m}$	Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Mass of Sun		$2.0 \times 10^{30} \text{ kg}$	Permittivity of free space	ϵ_0	$8.85 \times 10^{-12} \text{ F m}^{-1}$
1 AU		$1.5 \times 10^{11} \text{ m}$	Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$	Speed of light in vacuum	c	$3.0 \times 10^8 \text{ m s}^{-1}$
Universal constant of gravitation	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	Speed of sound in air	v	$3.4 \times 10^2 \text{ m s}^{-1}$

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

<i>Substance</i>	<i>Refractive index</i>	<i>Substance</i>	<i>Refractive index</i>
Diamond	2.42	Glycerol	1.47
Glass	1.51	Water	1.33
Ice	1.31	Air	1.00
Perspex	1.49	Magnesium Fluoride	1.38

SPECTRAL LINES

<i>Element</i>	<i>Wavelength/nm</i>	<i>Colour</i>	<i>Element</i>	<i>Wavelength/nm</i>	<i>Colour</i>
Hydrogen	656	Red	Cadmium	644	Red
	486	Blue-green		509	Green
	434	Blue-violet		480	Blue
	410	Violet	<i>Lasers</i>		
	397	Ultraviolet	<i>Element</i>	<i>Wavelength/nm</i>	<i>Colour</i>
	389	Ultraviolet	Carbon dioxide	9550 } 10590 }	Infrared
Sodium	589	Yellow	Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

<i>Substance</i>	<i>Density/ kg m⁻³</i>	<i>Melting Point/ K</i>	<i>Boiling Point/K</i>	<i>Specific Heat Capacity/ J kg⁻¹ K⁻¹</i>	<i>Specific Latent Heat of Fusion/ J kg⁻¹</i>	<i>Specific Latent Heat of Vaporisation/ J kg⁻¹</i>
Aluminium	2.70×10^3	933	2623	9.02×10^2	3.95×10^5
Copper	8.96×10^3	1357	2853	3.86×10^2	2.05×10^5
Glass	2.60×10^3	1400	6.70×10^2
Ice	9.20×10^2	273	2.10×10^3	3.34×10^5
Glycerol	1.26×10^3	291	563	2.43×10^3	1.81×10^5	8.30×10^5
Methanol	7.91×10^2	175	338	2.52×10^3	9.9×10^4	1.12×10^6
Sea Water	1.02×10^3	264	377	3.93×10^3
Water	1.00×10^3	273	373	4.19×10^3	3.34×10^5	2.26×10^6
Air	1.29
Hydrogen	9.0×10^{-2}	14	20	1.43×10^4	4.50×10^5
Nitrogen	1.25	63	77	1.04×10^3	2.00×10^5
Oxygen	1.43	55	90	9.18×10^2	2.40×10^4

The gas densities refer to a temperature of 273 K and a pressure of $1.01 \times 10^5 \text{ Pa}$.

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1. A flywheel consisting of a solid, uniform disc is free to rotate about a fixed axis as shown in Figure 1A. The disc has a mass of 16 kg and a radius of 0.30 m.

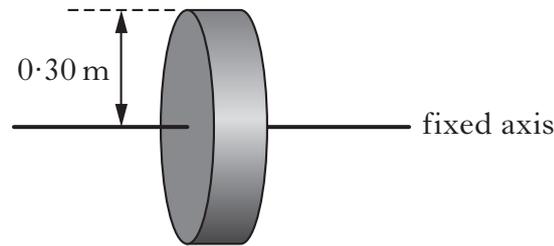


Figure 1A

- (a) Calculate the moment of inertia of the flywheel. 2
- (b) A mass is attached to the flywheel by a light string as shown in Figure 1B.

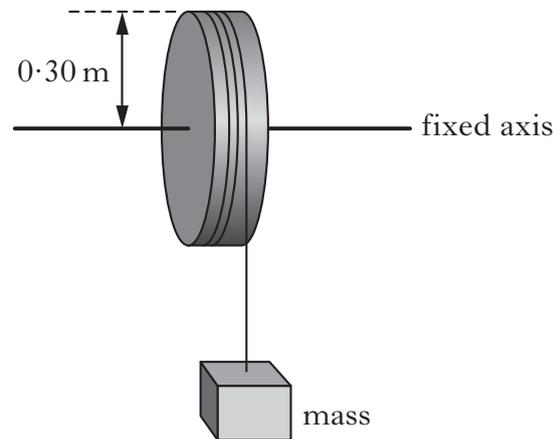


Figure 1B

The mass is allowed to fall and is found to be travelling at 3.0 m s^{-1} when the string leaves the flywheel. The flywheel makes 5 further revolutions before it comes to rest.

- (i) Calculate the angular acceleration of the flywheel after the string leaves the flywheel. 3
- (ii) Calculate the frictional torque acting on the flywheel. 2
- (c) The experiment is repeated with a flywheel made from a more dense material with the same physical dimensions. The string, falling mass and all frictional forces are the same as in part (b).

As the string detaches from the flywheel, is the speed of the falling mass greater than, the same as or less than 3.0 m s^{-1} ?

You must justify your answer. 2

1. (continued)

(d) A Kinetic Energy Recovery System (KERS) is used in racing cars to store energy that is usually lost when braking.

One of these systems uses a flywheel, as shown in Figure 1C, to store the energy.

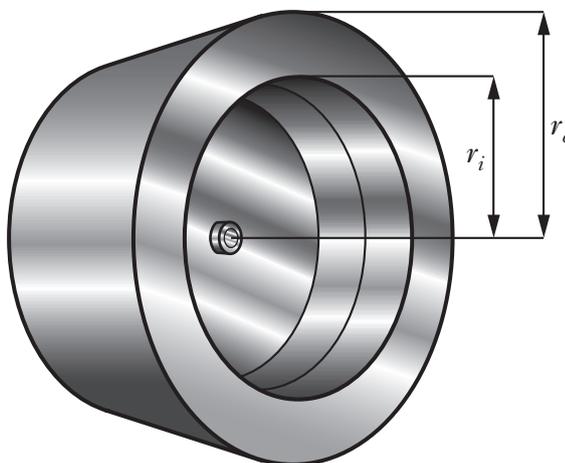


Figure 1C

Data for this KERS flywheel is given below.

Internal radius r_i = 0.15 m

External radius r_e = 0.20 m

Mass of flywheel M = 6.0 kg

Maximum rate of revolution = 6.0×10^4 revolutions per minute

(i) Using the expression

$$I = \frac{1}{2} M(r_i^2 + r_e^2)$$

determine the moment of inertia of the flywheel.

1

(ii) Calculate the maximum rotational kinetic energy that can be stored in the flywheel.

3

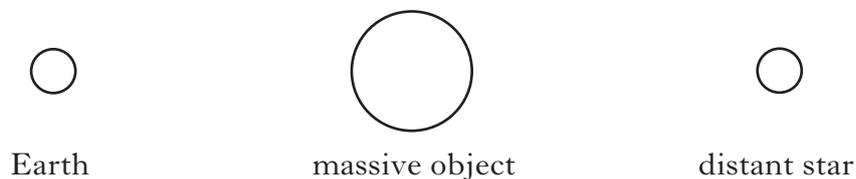
(13)

[Turn over

2. (a) With reference to General Relativity, explain why the Moon orbits the Earth. 2

(b) General Relativity also predicts gravitational lensing.

Figure 2 shows the relative positions of Earth, a massive object and a distant star.



Not to scale

Figure 2

Copy the diagram. On your diagram show:

- (i) the path of light from the star to Earth; 1
- (ii) the observed position of the star from Earth. 1

(c) Two students visit the tallest building on Earth. Student A takes a lift to the top of the building while student B waits at the bottom. General Relativity predicts that time will not pass at the same rate for both students. For which student does time pass at a slower rate?

You must justify your answer. 2

(6)

3. The luminosity of the Sun is 3.9×10^{26} W and the mean radius of the Earth's orbit around the Sun is 1 astronomical unit (AU).

(a) Calculate the Sun's apparent brightness at the surface of the Earth. 2

(b) The distance d to a star can be calculated using the relationship:

$$10^{0.2(m-M)} = \frac{d}{10}$$

This gives a distance in parsecs. 1 parsec is equivalent to 3.26 light years.

The apparent magnitude (m) of a celestial body is a measure of its brightness as viewed from Earth. The absolute magnitude (M) of a celestial body is a measure of its intrinsic brightness.

The following data was obtained for a star.

Apparent magnitude = 5.62

Absolute magnitude = -4.38

Calculate the distance in light years to this star from Earth. 3

(5)

4. The lyrics of the song Woodstock contain the lines "We are stardust; we are golden. We are billion year old carbon".

Use your knowledge of Physics to comment on these lyrics. (3)

[Turn over

5. A typical Hertzsprung-Russell (H-R) diagram is shown in Figure 5A.

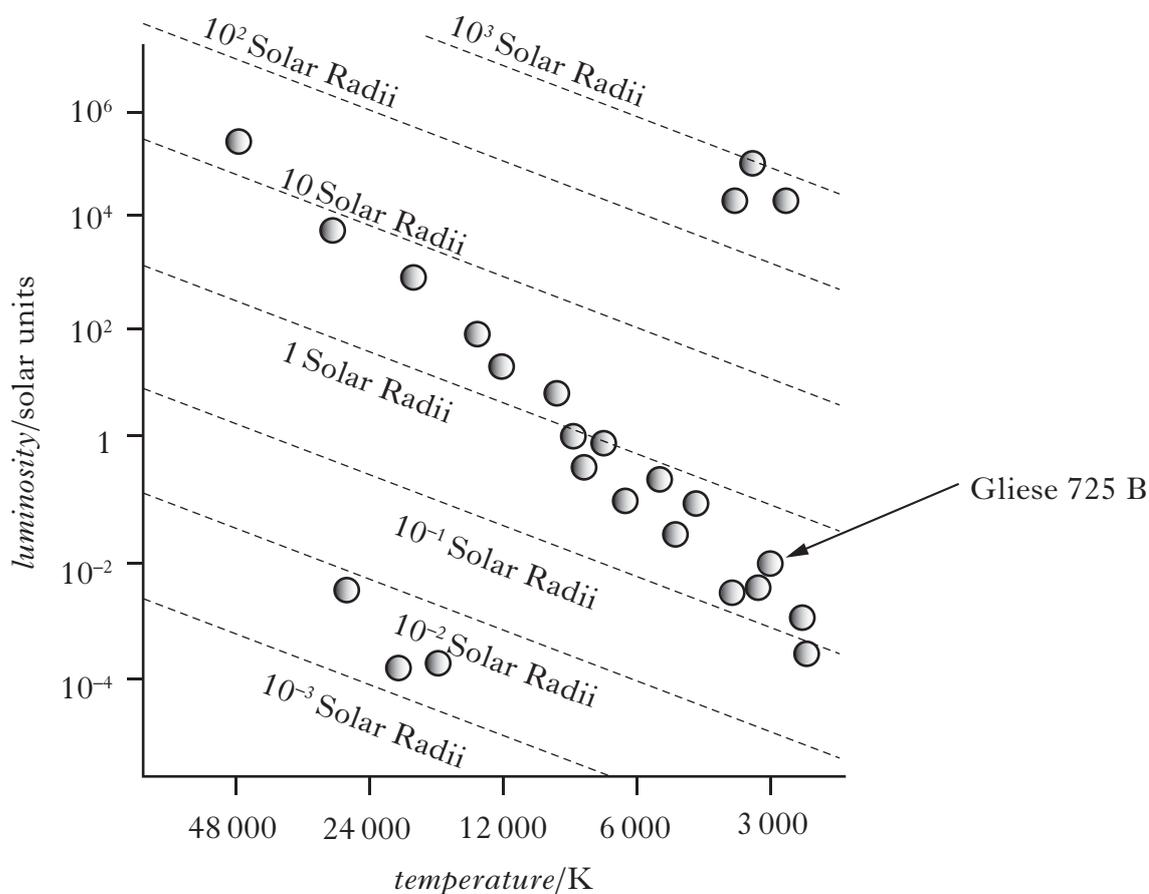


Figure 5A

(a) The luminosity of the Sun is 3.9×10^{26} W.

Using information from Figure 5A:

- (i) determine the luminosity in watts of Gliese 725 B; 1
- (ii) show that the radius of Gliese 725 B is 3×10^8 m; 2
- (iii) explain why it would be inappropriate to give the answer for part (ii) to more than one significant figure. 1

5. (continued)

- (b) Figure 5B shows how the radiation intensity varies with frequency for a black body radiator.

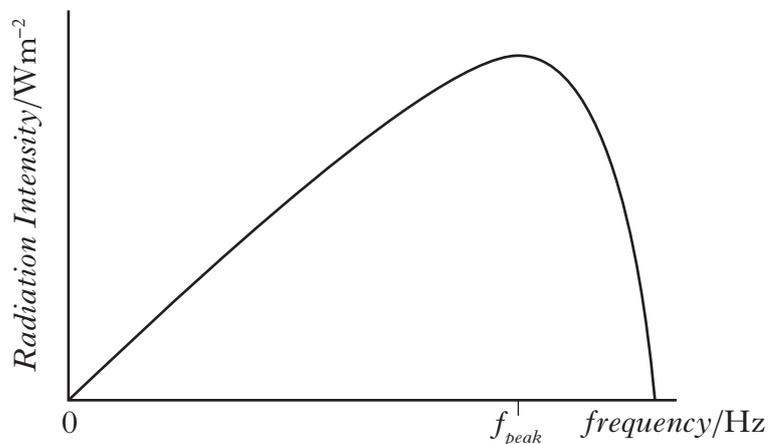


Figure 5B

This spectrum has a peak intensity at a frequency of f_{peak} .
 f_{peak} can be estimated using the relationship

$$f_{peak} = \frac{2.8k_bT}{h}$$

where $k_b = 1.38 \times 10^{-23} \text{ J K}^{-1}$ (Boltzmann constant) and the other symbols have their usual meanings.

- (i) Estimate f_{peak} for Gliese 725 B. 2
- (ii) The cosmic microwave background radiation (CMBR) has a spectrum which peaks at a wavelength of 1.9 mm. Calculate the temperature of the CMBR. 3
- (c) Some astronomers have suggested that primordial black holes of mass 1.0×10^{-10} solar masses could make up the dark matter in our galaxy. Determine the Schwarzschild radius of such a black hole. 3

(12)

[Turn over

6. (a) (i) State what is meant by *simple harmonic motion*. 1
- (ii) The displacement of an oscillating mass can be described by the expression

$$y = A\sin\omega t$$

where the symbols have their usual meanings.

Show that this mass exhibits simple harmonic motion (SHM). 2

- (iii) The displacement of an object exhibiting SHM can also be written as

$$y = A\cos\omega t$$

Identify the initial condition for which this equation would be used. 1

- (b) A mass attached to a spring is displaced from its equilibrium position and allowed to oscillate vertically. A motion sensor, connected to a computer, is placed below the mass as shown in Figure 6A.

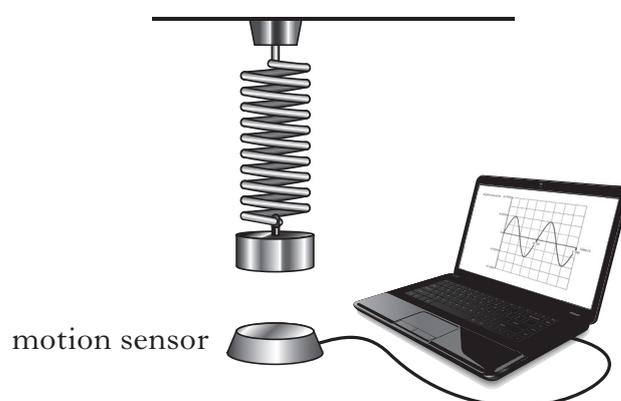


Figure 6A

Figure 6B shows the graph of the displacement from equilibrium position against time for the mass.

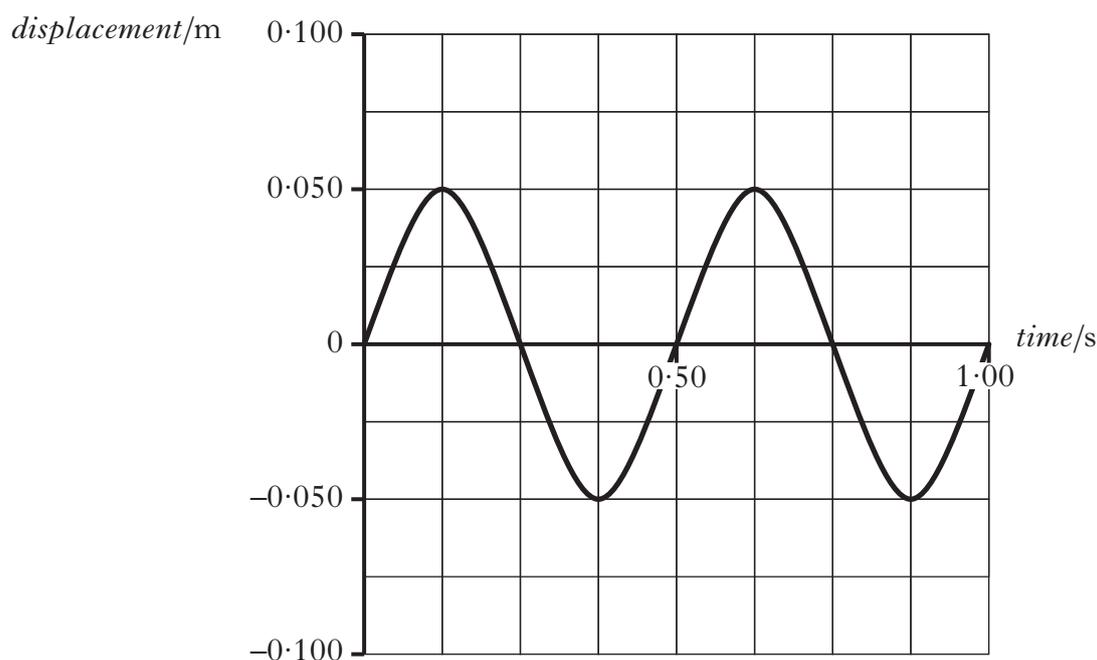


Figure 6B

6 (b) (continued)

- (i) Using data from the graph, determine the velocity of the mass at 0.50 s. **3**
- (ii) Calculate the maximum acceleration of the mass. **2**
- (c) The system is modified by attaching a rigid card of negligible mass as shown in Figure 6C.

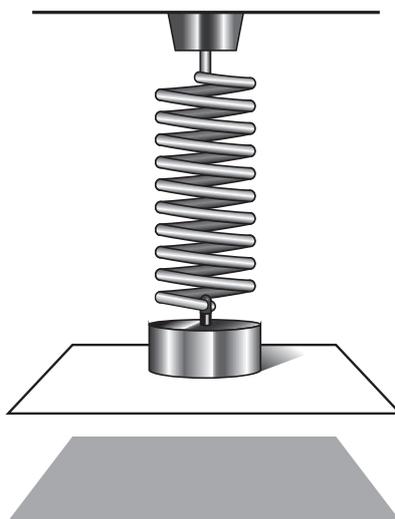


Figure 6C

The mass is displaced from its equilibrium position and allowed to oscillate vertically.

Sketch a displacement time graph of this motion.

1
(10)

[Turn over

7. One of the key ideas in Quantum Theory is the Heisenberg Uncertainty Principle.

(a) The uncertainty in the position of a particle can be estimated as its de Broglie wavelength. An electron has an average speed of $3.2 \times 10^6 \text{ m s}^{-1}$.

(i) Calculate the minimum uncertainty in the momentum of this electron. **3**

(ii) It is not possible to measure accurately the position of an electron using visible light. Describe the effect of using a beam of X-rays rather than visible light on the measurement of the electron's position and momentum. Justify your answer. **2**

(b) Polonium 212 decays by alpha emission. The energy required for an alpha particle to escape from the Polonium nucleus is 26 MeV. Prior to emission, alpha particles in the nucleus have an energy of 8.78 MeV. With reference to the Uncertainty Principle, explain how this process can occur. **2**

(7)

8. A student carries out a Young's double slit experiment in order to determine the wavelength of monochromatic red light.

The student uses the apparatus shown in Figure 8 to produce an interference pattern on the screen.

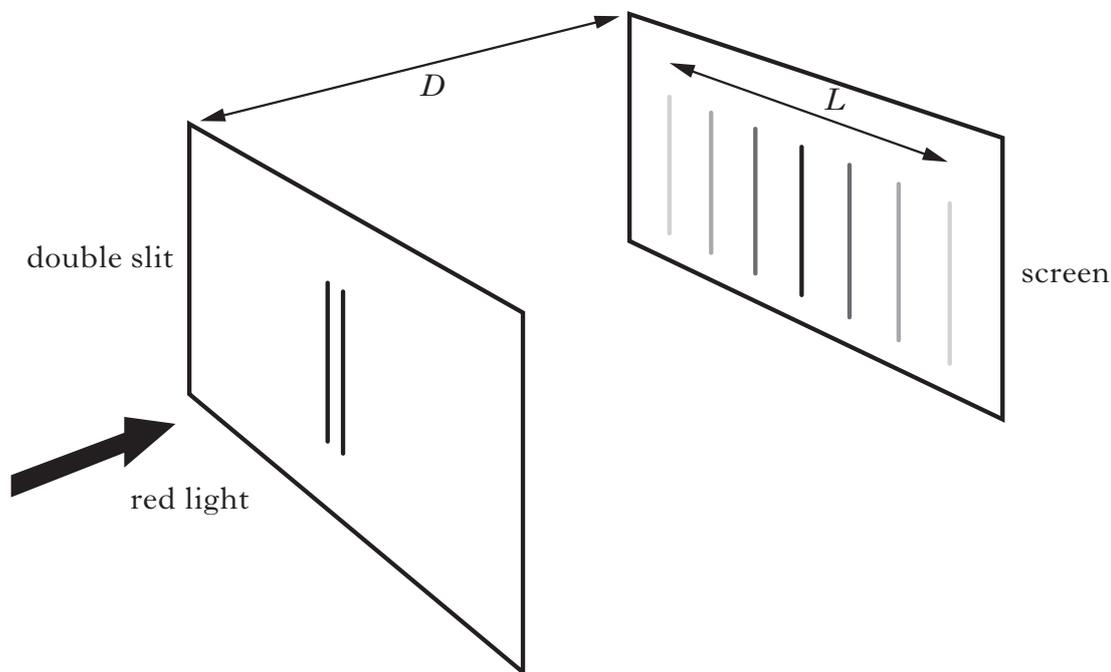


Figure 8

The double slit separation d is measured using a travelling microscope. The distance D between the double slit and the screen is measured using a steel measuring tape. The length L of the interference pattern is measured using a plastic ruler.

The student records the following data.

$$D = (4.250 \pm 0.005) \text{ m}$$

$$L = (67 \pm 2) \text{ mm}$$

$$d = (0.25 \pm 0.01) \text{ mm}$$

- (a) (i) State why it is possible to produce an interference pattern using only a single light source. 1
- (ii) Calculate the wavelength of the light from the source. 3
- (iii) Calculate the absolute uncertainty in the wavelength. 3
- (b) The student repeats the experiment with the same apparatus but uses a monochromatic blue light source. D remains fixed.
- State the effect this will have on the percentage uncertainty in the calculated value for the wavelength of the blue light.

You must justify your answer.

2

(9)

9. (a) A proton moving at constant speed v enters a uniform magnetic field of induction B as shown in Figure 9A.

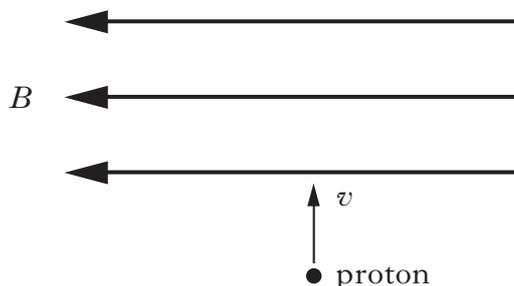


Figure 9A

Within the field the proton follows a circular path of radius r .

- (i) Explain why the proton follows a circular path. 1
- (ii) Show that the radius of the path r is given by

$$r = \frac{1.05 \times 10^{-8} v}{B}. \quad 2$$

- (b) Another proton moving at the same speed v enters the magnetic field at an angle θ to the magnetic field lines as shown in Figure 9B.

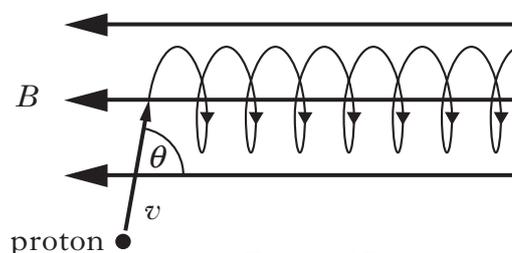


Figure 9B

Explain the shape of the path followed by this proton in the magnetic field. 2

9. (continued)

- (c) The solar wind is a stream of charged particles, mainly protons and electrons, released from the atmosphere of the Sun. Many of these particles become trapped by the magnetic field of the Earth.

Some of the trapped particles move back and forth in helical paths between two **magnetic mirror points**. The path followed by one particular proton is shown in Figure 9C.

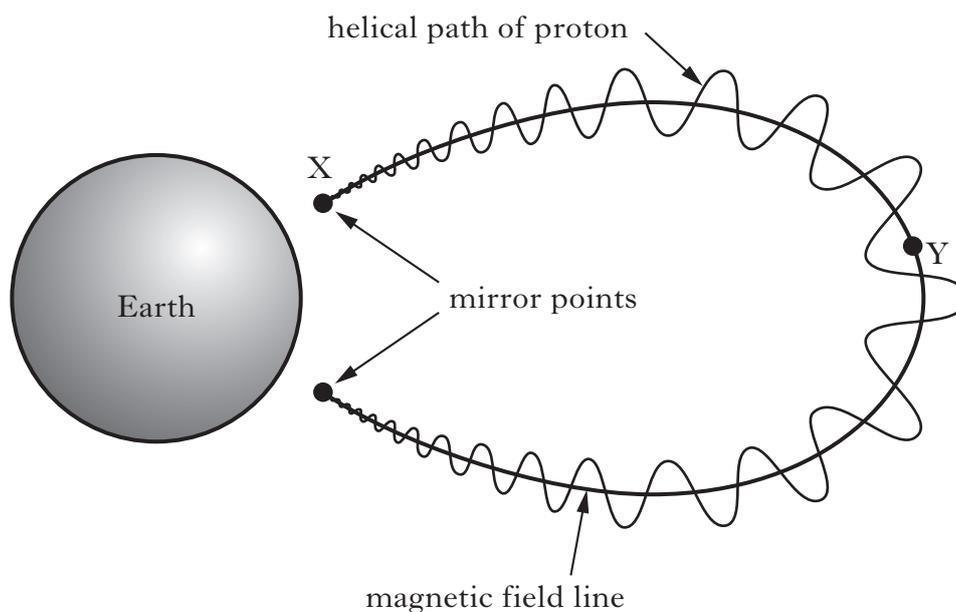


Figure 9C

The speed of the proton remains constant at $1.2 \times 10^7 \text{ m s}^{-1}$ as it travels along its helical path from one magnetic mirror point to the other.

- (i) The proton oscillates between the two mirror points with a frequency of 4.0 Hz. Calculate the distance that the proton travels in moving from one mirror point to the other. 3
- (ii) Explain why the radius of the helical path followed by the proton increases as it moves from point X to point Y as shown in Figure 9C. 1
- (iii) At point X the radius of curvature of the helix for this proton is $1.0 \times 10^4 \text{ m}$. Calculate the strength of the Earth's magnetic field at this point. 2

(11)

[Turn over

10. (a) A teacher investigates the electric field between two parallel metal plates X and Y using the apparatus shown in Figure 10A.

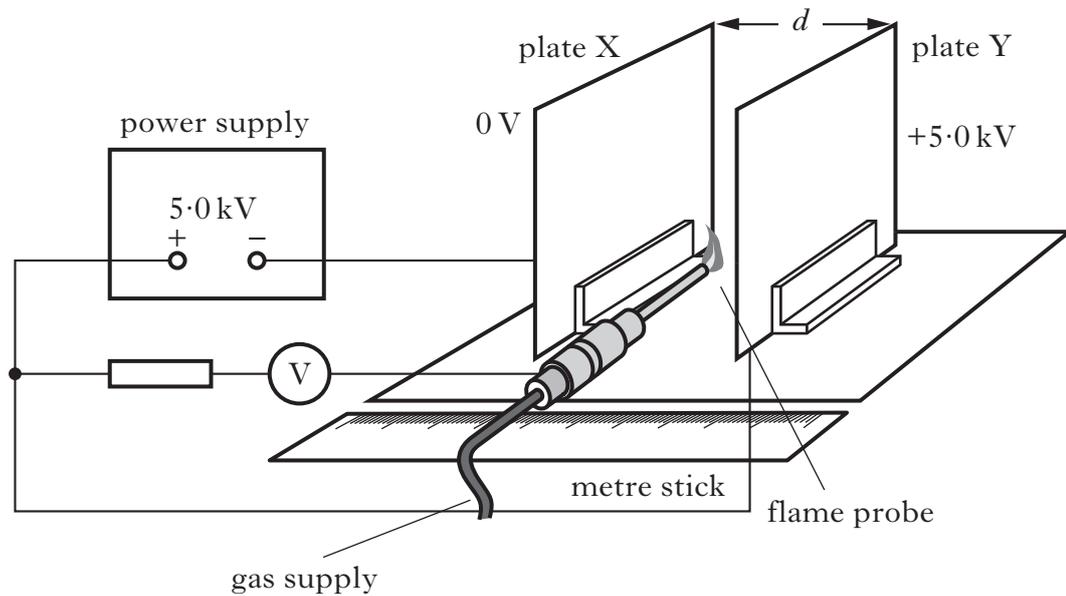


Figure 10A

The plates are connected to a 5.0 kV supply and are separated by a distance d .

A calibrated flame probe and voltmeter measure the potential relative to plate X. The probe is placed at different points between the plates. The distance from plate X and the potential at each point are measured.

The results are used to plot the graph shown in Figure 10B.

10. (a) (continued)

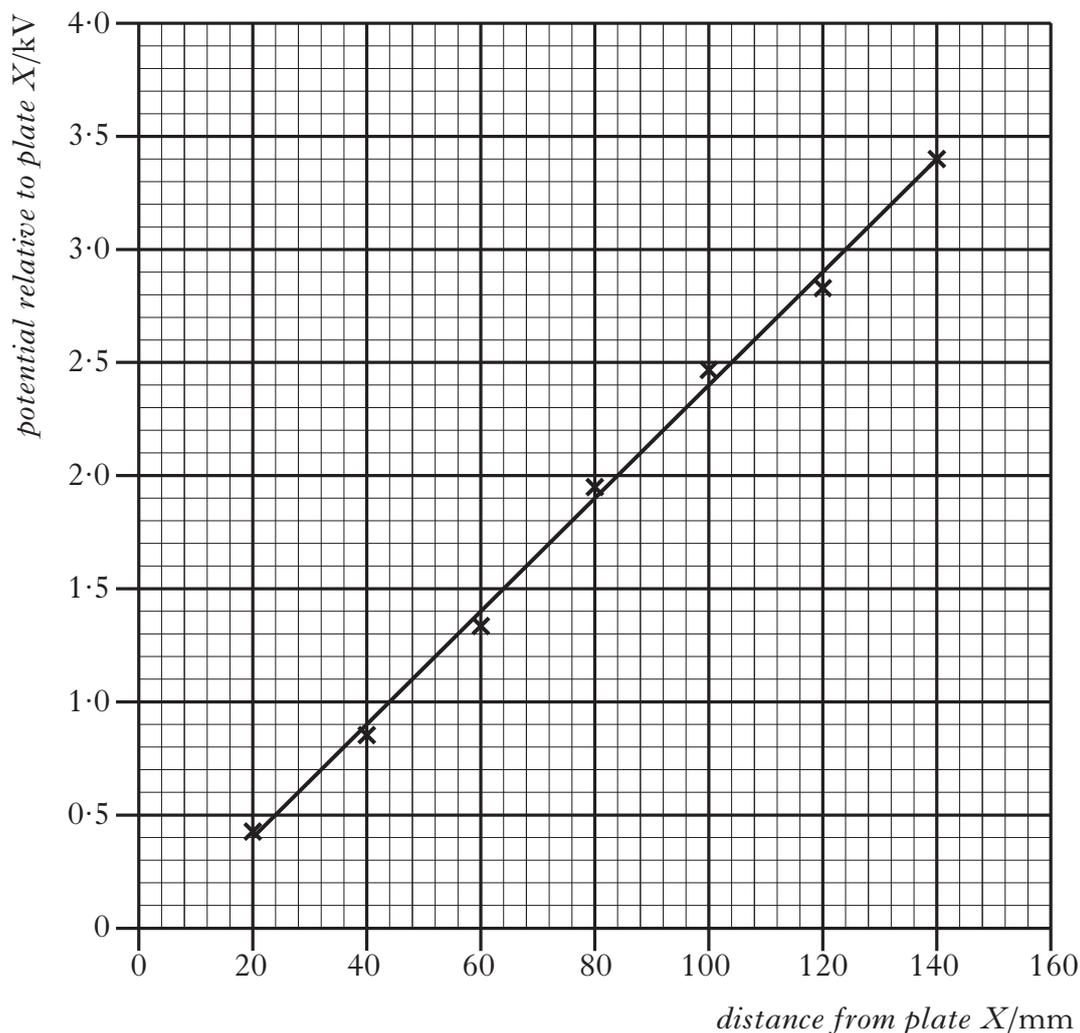


Figure 10B

- (i) The electric field strength in the region between the plates is considered to be uniform. Explain the meaning of the term *uniform electric field*. 1
- (ii) Using information from the graph, determine the electric field strength between the plates. 2
- (iii) Calculate the separation d of the plates. 2
- (iv) In theory the best fit line for this graph should pass through the origin. Suggest why the line on the graph in Figure 10B does not pass through the origin. 1

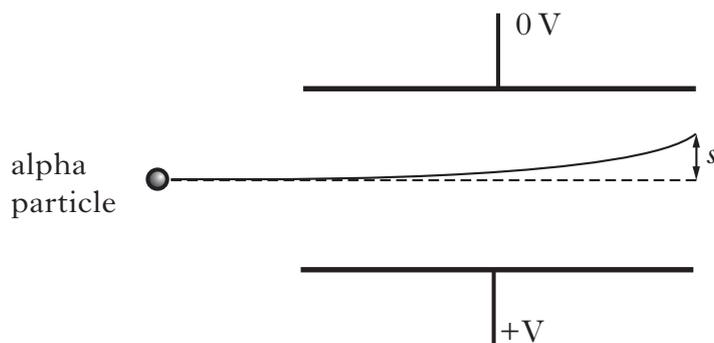
[Turn over

10. (continued)

- (b) In an experiment to investigate the deflection of alpha particles in an electric field a potential difference is applied across two parallel metal plates.

An alpha particle moving horizontally enters the region between the plates.

The alpha particle is deflected vertically by a distance s as shown in Figure 10C.



Not to scale

Figure 10C

The separation of the parallel plates is now increased. An alpha particle enters the electric field at the same point and with the same velocity as before.

What effect does this have on the magnitude of the deflection s ?

You must justify your answer.

2

(8)

11. A geomagnetic reversal is a change in polarity of the Earth's magnetic field. On average this happens every 300 000 years. Reversals can take in excess of 1000 years to complete. During a previous reversal, the strength of the Earth's magnetic field dropped to 5% of its present value.

Figure 11 shows a computer simulation of the Earth's magnetic field during a reversal.

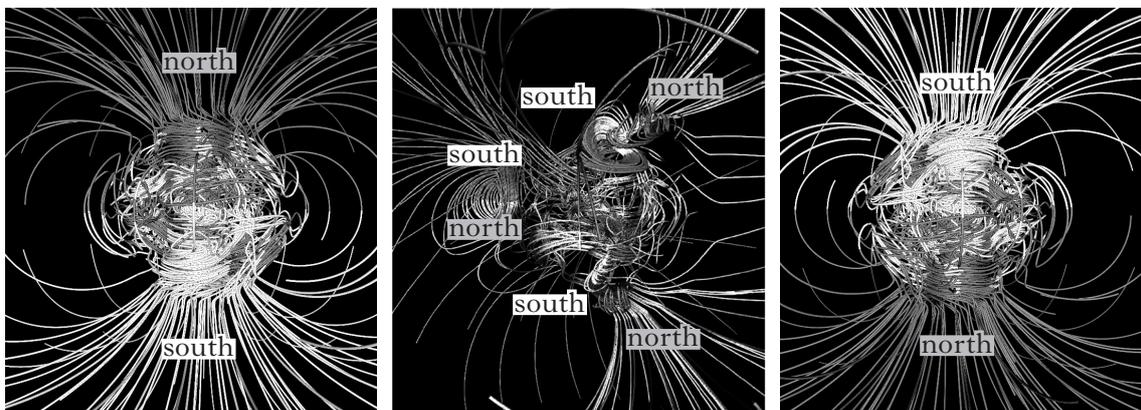


Figure 11

Use your knowledge of Physics to comment on the possible effects of such a reversal.

(3)

[Turn over

12. (a) A student investigates how the current in an inductor varies with the frequency of a voltage supply.

(i) Draw a suitable labelled circuit diagram of the apparatus required to carry out the investigation. 2

(ii) The student collects the following data.

<i>Frequency/Hz</i>	40	60	80	100	120
<i>Current/mA</i>	148	101	76.0	58.2	50.0

Determine the relationship between the supply frequency and current for this inductor. 2

(b) An inductor of inductance 3.0 H and negligible resistance is connected in a circuit with a $12\ \Omega$ resistor and supply voltage V_s as shown in Figure 12A.

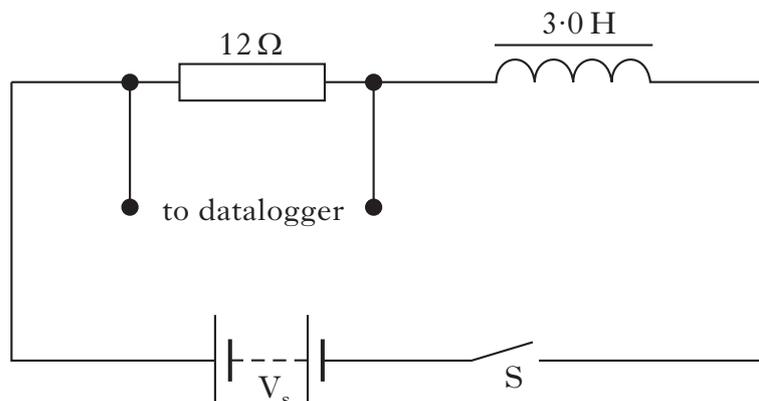


Figure 12A

The datalogger is set to calculate the back emf across the inductor.

Switch S is initially open.

Switch S is now closed. Figure 12B shows how the back emf across the inductor varies from the instant the switch is closed.

12. (b) (continued)

Marks

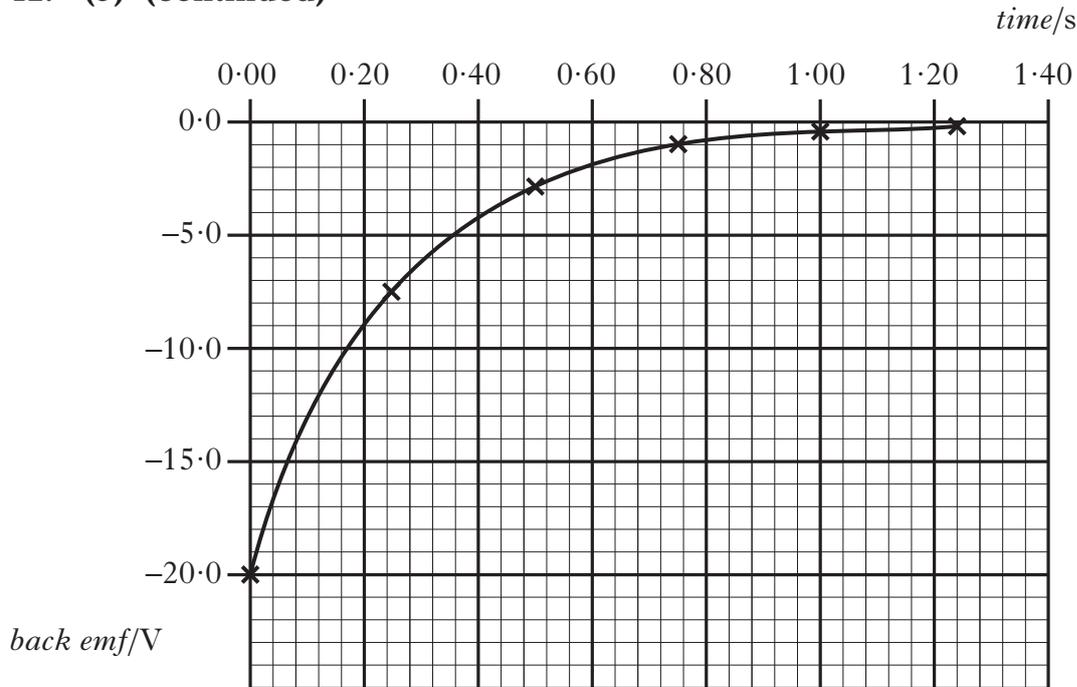


Figure 12B

- (i) Determine the voltage across the resistor at $t = 0.20$ s. 2
 - (ii) Calculate the rate of change of current in the circuit at $t = 0.40$ s. 2
 - (iii) State why the magnitude of the back emf is greatest at $t = 0$. 1
- (c) A tuned circuit consisting of an inductor, capacitor and resistor is shown in Figure 12C.

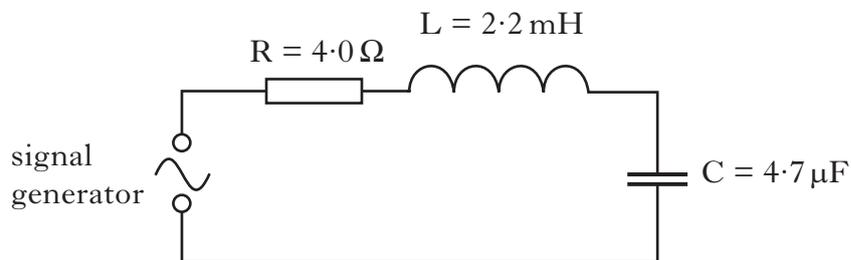


Figure 12C

The impedance Z , measured in ohms, of the circuit is given by the relationship

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where the symbols have their usual meanings.

- (i) At a particular frequency f_0 , the impedance of the circuit is a minimum.

Show that f_0 is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}. \quad 1$$

- (ii) Calculate the frequency f_0 . 2
- (iii) State the minimum impedance of the circuit. 1

(13)

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ACKNOWLEDGEMENTS

Question 4 – Lyrics – Three lines are taken from “*Woodstock*” by Joni Mitchell.

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Question 6(b) Fig 6A – [cobalt88/shutterstock.com](https://www.shutterstock.com/user/cobalt88)

Question 11 – Images of geomagnetic reversal. Reproduced by kind permission of Professor Gary A Glatzmaier, University of California, Santa Cruz.

X272/13/01

NATIONAL
QUALIFICATIONS
2015

TUESDAY, 5 MAY
1.00 PM – 3.30 PM

PHYSICS
ADVANCED HIGHER
(Revised)

Relationships required for Advanced Higher Physics (Revised)

(For reference, relationships required for Higher Physics (Revised) are also included on *Page four*)

Relationships required for Physics Advanced Higher (Revised)

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$F = \frac{mv^2}{r} = mr\omega^2$$

$$T = Fr$$

$$T = I\alpha$$

$$L = mvr = mr^2\omega$$

$$L = I\omega$$

$$E_k = \frac{1}{2}I\omega^2$$

$$F = G \frac{Mm}{r^2}$$

$$V = -\frac{GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

apparent brightness, $b = \frac{L}{4\pi r^2}$

Power per unit area = σT^4

$$L = 4\pi r^2 \sigma T^4$$

$$r_{\text{Schwarzschild}} = \frac{2GM}{c^2}$$

$$E = hf$$

$$\lambda = \frac{h}{p}$$

$$mvr = \frac{nh}{2\pi}$$

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$F = qvB$$

$$\omega = 2\pi f$$

$$a = \frac{d^2y}{dt^2} = -\omega^2 y$$

$$y = A \cos \omega t \quad \text{or} \quad y = A \sin \omega t$$

$$v = \pm \omega \sqrt{(A^2 - y^2)}$$

$$E_K = \frac{1}{2} m \omega^2 (A^2 - y^2)$$

$$E_P = \frac{1}{2} m \omega^2 y^2$$

$$y = A \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$$

$$\phi = \frac{2\pi x}{\lambda}$$

$$\text{optical path difference} = m\lambda \quad \text{or} \quad \left(m + \frac{1}{2} \right) \lambda$$

where $m = 0, 1, 2, \dots$

$$\Delta x = \frac{\lambda l}{2d}$$

$$d = \frac{\lambda}{4n}$$

$$\Delta x = \frac{\lambda D}{d}$$

$$n = \tan i_p$$

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi \epsilon_0 r}$$

$$F = QE$$

$$V = Ed$$

$$F = IlB \sin \theta$$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$t = RC$$

$$X_C = \frac{V}{I}$$

$$X_C = \frac{1}{2\pi f C}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$E = \frac{1}{2} LI^2$$

$$X_L = \frac{V}{I}$$

$$X_L = 2\pi f L$$

$$\frac{\Delta W}{W} = \sqrt{\left(\frac{\Delta X}{X} \right)^2 + \left(\frac{\Delta Y}{Y} \right)^2 + \left(\frac{\Delta Z}{Z} \right)^2}$$

$$\Delta W = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2}$$

Relationships required for Physics Higher (Revised)

$$d = \bar{v}t$$

$$E_W = QV$$

$$V_{peak} = \sqrt{2}V_{rms}$$

$$s = \bar{v}t$$

$$E = mc^2$$

$$I_{peak} = \sqrt{2}I_{rms}$$

$$v = u + at$$

$$E = hf$$

$$Q = It$$

$$s = ut + \frac{1}{2}at^2$$

$$E_K = hf - hf_0$$

$$V = IR$$

$$v^2 = u^2 + 2as$$

$$E_2 - E_1 = hf$$

$$P = IV = I^2R = \frac{V^2}{R}$$

$$s = \frac{1}{2}(u + v)t$$

$$T = \frac{1}{f}$$

$$R_T = R_1 + R_2 + \dots$$

$$W = mg$$

$$v = f\lambda$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

$$F = ma$$

$$d\sin\theta = m\lambda$$

$$E = V + Ir$$

$$E_W = Fd$$

$$n = \frac{\sin\theta_1}{\sin\theta_2}$$

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right)V_S$$

$$E_P = mgh$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

$$E_K = \frac{1}{2}mv^2$$

$$\sin\theta_c = \frac{1}{n}$$

$$C = \frac{Q}{V}$$

$$P = \frac{E}{t}$$

$$I = \frac{k}{d^2}$$

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

$$p = mv$$

$$I = \frac{P}{A}$$

$$Ft = mv - mu$$

$$\text{path difference} = m\lambda \quad \text{or} \quad \left(m + \frac{1}{2}\right)\lambda \quad \text{where } m = 0, 1, 2, \dots$$

$$F = G\frac{Mm}{r^2}$$

$$\text{random uncertainty} = \frac{\text{max. value} - \text{min. value}}{\text{number of values}}$$

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$l' = l\sqrt{1 - \left(\frac{v}{c}\right)^2}$$

$$f_o = f_s \left(\frac{v}{v \pm v_s}\right)$$

$$z = \frac{\lambda_{observed} - \lambda_{rest}}{\lambda_{rest}}$$

$$z = \frac{v}{c}$$

$$v = H_0d$$